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THE REDUNDANCY OF MATHEMATICS INSTRUCTION IN U.S. ELEMENTARY AND MIDDLE SCHOOLS

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ABSTRACT
International comparisons have highlighted that the U.S. mathematics curriculum, both in terms of curriculum influences (e.g., textbooks, standards) and actual instruction, is broad and shallow. Standards-based reform is explicitly designed to improve coherence and reduce redundancy across grades. This article evaluates the redundancy of mathematics instruction after the early years of standards-based reform, using survey data from over 7,000 teachers. Instruction for teachers in consecutive grades in the same school is compared, and the redundancy of instruction is compared with the redundancy of state standards. Results indicate continuing problems of instructional redundancy, with upwards of 60% of instructional time at each grade on content taught in previous grades. Topics are continually introduced to the curriculum but leave more slowly; middle school instruction is broader and more redundant than elementary school instruction. State standards are also highly redundant, though there is no apparent relationship between the redundancy of state standards and the redundancy of instruction.

It is by now a cliché to point out that the U.S. mathematics curriculum is like the Missouri River, a mile wide and an inch deep (e.g., Schmidt et al., 2001). This appears to be true both for the intended and enacted curricula as reported by teachers (Porter, 1989; Schmidt et al., 2001; Schmidt, McKnight, & Raizen, 1997; Schmidt, Wang, & McKnight, 2005). Textbooks and content expectations in U.S.

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mathematics push toward curriculum coverage that is broad (i.e., covering many topics at each grade) and shallow (i.e., covering topics at a superficial level of detail). The shallowness and overwhelming breadth of the curriculum is believed to contribute to U.S. students' relatively low levels of conceptual understanding in mathematics and their dissatisfaction with and lack of interest in mathematics (Schmidt et al., 2001).

A close corollary to the breadth and shallowness of the U.S. mathematics curriculum is its remarkable redundancy, especially in the early and middle grades. Again, research suggests that redundancy characterizes both the intended and enacted curriculum as reported by teachers (Flanders, 1987; Hiebert et al., 2005; Porter, 1989; Porter, Polikoff, & Smithson, 2009; Schmidt et al., 1997, 2001, 2005). Content standards documents in the United States tend to introduce topics at the same rate as curriculum frameworks from other countries do, but the topics tend to linger in the U.S. curricula for many grades (Porter et al., 2009; Schmidt et al., 1997, 2001, 2005). Textbooks also contribute, emphasizing the same content in grade after grade (Flanders, 1987; Schmidt et al., 2001). And limited evidence about teachers' reported instruction indicates that the redundancy emphasized in the various curriculum influences contributes to a repetitive curriculum in which topics are introduced, taught shallowly, and reintroduced in later grades (Porter, 1989; Schmidt et al., 2001).

To be sure, some degree of redundancy is to be expected as topics are reintroduced and taught at increasing levels of cognitive complexity. But the limited evidence suggests that mathematics education in the United States is highly redundant when compared against international benchmarks (Schmidt et al., 2005) and stated views of scholars (Porter, 1989; Schmidt et al., 2001) and professional organizations (National Council of Teachers of Mathematics, 1989, 2006). The redundancy not only takes up valuable instructional time, but it also contradicts the key underlying goal of standards-based reform—increased curriculum coherence.

While previous literature on the redundancy of the mathematics curriculum is instructive, it is limited in several important regards. First, most of the literature predates No Child Left Behind (NCLB) and the strengthening of standards-based educational reforms in the 2000s. Second, most previous research uses curriculum data at too coarse a grain size to accurately portray redundancy and account for the possibility of spiraling. Third, the degree or nature of the redundancy has generally not been described in detail. Fourth, little of the previous research has considered teachers at adjacent grades and in different academic tracks.

This article contributes to an understanding of the extent to which the United States' intended and enacted mathematics curricula remain highly repetitive. Teacher survey data from 7,366 teachers in 1,049 schools in 27 states and content analyses of mathematics content standards in 17 states and the Common Core State Standards are used to investigate four research questions: (1) How much redundancy is there in mathematics instruction between grades in mathematics grades K–8? (2) To what extent does redundancy in middle school differ based on course sequencing in middle school (e.g., pre-algebra in seventh grade, eighth grade, or later)? (3) What is the main new and redundant content in each grade's mathematics instruction, K–8? (4) To what extent do state K–8 content standards and the Common Core State Standards in mathematics emphasize redundant content, and how is redundancy in state standards related to redundancy in the enacted curriculum?

The research questions are motivated by a desire to understand the current state of mathematics instruction in U.S. schools after a decade or more of standards-based reforms intended to improve curriculum coherence. Understanding the most common new and redundant topics across grades might also help identify the major culprits contributing to redundancy and, therefore, reveal prime targets for instructional improvement. Finally, understanding the extent to which state and Common Core Standards are redundant should offer a sense of how these documents have contributed and the likelihood that they will continue to contribute to a reduction in instructional redundancy.

Background

The focus of this research is the enacted curriculum of target classes, or the particular content covered by the teacher in those classes (Porter & Smithson, 2001). The enacted curriculum has been a target of education policy for decades (Smith & O'Day, 1991), both because the content of instruction is predictive of student learning (e.g., Gamoran, Porter, Smithson, & White, 1997; McKnight et al., 1987; Schmidt, 1983; Sebring, 1987) and also because the content of instruction is seen as being directly manipulable by policy (Smith & O'Day, 1991). Under standards-based educational reforms such as the No Child Left Behind Act, the enacted curriculum is the key mediating variable separating educational policies from student achievement (Clune, 1993; Smith & O'Day, 1991).

The idea of a coherent sequence of curriculum experiences in grades K–12 is an explicit purpose of the establishment of content standards. This is especially true with the recent push toward college and career readiness standards and the Common Core, where standards have been developed by starting from the key skills students are to know by the time they graduate high school and structuring the curriculum in such a way as to reach that content goal.

Redundancy in a Spiraled Curriculum

Complicating the desire for a more streamlined, coherent sequence of content with less redundancy is the lack of agreement about the most effective curriculum structure for improving student understanding. In broad terms, at one end of the spectrum is a fully spiraled curriculum, in which concepts are continually reintroduced but at increasing levels of cognitive challenge (Bruner, 1960). Each reintroduction of a concept in a spiraled curriculum should increase student understanding of that concept. At the other end of the spectrum is a strand-based curriculum, in which fewer topics are studied over a longer period of time, and student mastery of the content in one strand leads to new strands. A review of the pros and cons to each approach is outside the scope of this article; however, it is important to note that there are different approaches to the structure of content in a mathematics curriculum. Obviously, in a spiraled curriculum there should be a greater degree of redundancy. Even in such a curriculum, however, the goal would not be successive reintroduction of the same skills, but rather successively introducing the same or similar topics but at increasingly challenging levels of cognitive demand.

Studies of Curriculum Redundancy

A number of studies of the redundancy of both the intended and enacted curriculum in U.S. K–12 mathematics exist (Flanders, 1987; Good, Grouws, & Ebmeier, 1983; Hiebert et al., 2005; McKnight et al., 1987; Porter, 1989; Porter et al., 2009; Schmidt et al., 1997, 2001, 2005). These studies all highlight that the level of redundancy of curriculum materials, content standards, and instruction is high across years in mathematics, and especially high relative to the level of redundancy in other countries.

Perhaps the most distal influences on the content of classroom instruction are content standards, created and promulgated by states and the National Council of Teachers of Mathematics (NCTM). Domestic and comparative international studies highlight that both state and NCTM standards tend to contain long lists of topics that remain in the curriculum for as many as eight grades (Porter et al., 2009; Schmidt et al., 1997, 2001, 2005). In contrast, standards documents from higher-achieving countries tend to cover topics for only 2 or 3 years before moving on to new topics (Schmidt et al., 2005). These results indicate that U.S. content standards are often overwhelmingly diffuse, based on an apparent organizing principle of including almost every topic at every grade (Schmidt et al., 2005, p. 556). To be sure, the NCTM standards are presented in grade bands rather than in grade-specific fashion, making the interpretation of redundancy less clear. Perhaps more appropriate would be an analysis of the NCTM's more recent Focal Points document (National Council of Teachers of Mathematics, 2006), which offers grade-by-grade guidance, but no such research yet exists.

More proximal to instructional choices than content standards are textbooks and other curriculum materials. Studies of mathematics curriculum materials dating back to the 1980s indicate high degrees of redundancy, especially in the middle school grades (Flanders, 1987). More recent studies using the Third International Math and Science Study (TIMSS) data (Schmidt et al., 1997, 2001) have supported these conclusions: at all grades, U.S. textbooks cover almost all the topics that are taught across grades K–8, many more topics than are covered in most other countries' textbooks. Again, in other countries, fewer topics are covered at each grade, and topics enter and exit the textbooks over the grade progression.

Finally, the few available studies of the enacted curriculum—teachers' instruction—indicate that redundancy is again a defining characteristic. In an early study of 34 Michigan fourth- and fifth-grade mathematics teachers using instructional logs (Porter, 1989), less than 10% of the variation in instructional coverage of particular topics was accounted for by grade level; teachers' coverage of topics was mostly idiosyncratic to each teacher rather than attributable to the grade in which they taught. Results from the TIMSS corroborated these findings, with teacher survey data indicating that instruction at eighth grade was highly broad, covering many of the same topics as in seventh and earlier grades (Schmidt et al., 1997, 2001). Furthermore, classroom observations suggested that a large proportion of the redundancy of the mathematics enacted curriculum was due to repeated, heavy emphasis on review of previously taught material—more than half of instructional time in each lesson (Hiebert et al., 2005).

The results of analyses of standards, textbooks, and classroom instruction all support the conclusion that the mathematics curriculum in the United States is diffuse and repetitive. Topics enter the curriculum and linger for many grades before exiting, leading to instruction that focuses on many topics shallowly. While the results of prior research are instructive, the utility of the findings for current thinking about curriculum policy are limited in several ways. For one, most of the studies predate the national implementation of standards-based reform under the No Child Left Behind Act. Standards-based reform is an educational policy system explicitly designed to improve the coherence of the curriculum, in part through a delineation of clear, focused content expectations (Smith & O'Day, 1991). If standards-based reform were effective, and the content standards specified nonrepetitive, coherent content, instruction should follow.

A second limitation is that many of the frameworks used in previous research on the redundancy of the curriculum included only a small list of instructional topics. For instance, the TIMSS work contains 32 topics and ignores cognitive demand (Schmidt et al., 2005). These lists of topics are limited in two ways: first, lists that ignore level of cognitive demand cannot account for the fact that a spiraling curriculum could return to the same topics but emphasize increasing levels of cognitive demand; second, there are many subtopics underneath each of the topics in the TIMSS framework (e.g., polygons and circles, which could have subtopics of area, perimeter, regular polygons, etc.), so it is possible that the degree of redundancy is inflated by the small number of topics listed.

Data and Method

The Surveys of Enacted Curriculum

The research presented here is a secondary analysis of teacher-survey and content-analysis data based on the Surveys of Enacted Curriculum (SEC) content taxonomy in mathematics. The origins of the SEC work are in efforts to understand teachers' decisions about what content to teach. A full history of the line of research is available elsewhere (Porter, 2002; Porter, Floden, Freeman, Schmidt, & Schwille, 1988). The SEC frameworks, including lists of topics, were developed with the input of educational professionals and through analysis of curriculum documents. The present form of the SEC began wide-scale use in the early 2000s.

The content taxonomy for the mathematics SEC has two dimensions, defining content at the intersection of 183 specific topics and five levels of cognitive demand. The coarse-grained topics are listed in Table 1, and the cognitive demand levels are in Table 2; the full list of topics is available in the survey online. While the list of topics is intended to be exhaustive, the SEC does not differentiate content based on complexity in mathematical topics (e.g., a lesson on dividing fractions with one-digit numbers in the denominators and a lesson on dividing fractions with three-digit numbers in the denominators would both be classified under "divide fractions"). In SEC language, the intersection of a particular topic with a particular level of cognitive demand is called a *cell*; there are 915 cells in the SEC mathematics taxonomy.

Teacher surveys. When teachers complete the SEC, they are provided with written instructions but may or may not receive training depending on the group leading the data collection. Teachers are instructed to think about a "target class" and a particular time period. Generally, teachers are instructed to choose a particular target class (e.g., the class they meet with third in a typical week). The time period is most

Table 1. List of Coarse-Grained Topics on the SEC

Coarse-Grained Topics on the SEC
Number sense / properties / relationships
Operations
Measurement
Consumer applications
Basic algebra
Advanced algebra
Geometric concepts
Advanced geometry
Data displays
Statistics
Probability
Analysis
Trigonometry
Special topics
Functions
Instructional technology

Note.—The full list of subtopics is available on the SEC online survey.

often a semester or a full academic year. The results are always aggregated up to represent a full year's instruction; for instance, if a teacher fills out the SEC once in the winter and once in the summer, the results from each survey are added together, weighted by the number of instructional days measured on each survey.

For each topic, teachers first indicate whether or not it was covered at all in the particular time period. For each topic covered, teachers then indicate how much time they spent on that topic using a scale of 1 = slight coverage (less than one class/lesson), 2 = moderate coverage (one to five classes/lessons), and 3 = sustained coverage (more than five classes/lessons). Finally, teachers divide the instructional emphasis for each topic among the five levels of cognitive demand using a scale of 0 = no emphasis (not an expectation for this topic), 1 = slight emphasis (accounts for less than 25% of the time spent on this topic), 2 = moderate emphasis (accounts for 25% to 33% of the time spent on this topic), and 3 = sustained emphasis (accounts for more than 33% of the time spent on this topic). The data are turned into proportions of the year's instructional time. Thus, while it is possible that there may be an over-

Table 2. List of Cognitive Demand Levels and Example Activities on the SEC

Cognitive Demand Level	Example Activity 1	Example Activity 2
Memorize	Recite basic math facts	Recall math terms and definitions
Perform procedures	Do computational procedures or algorithms	Solve equations/formulas or routine word problems
Demonstrate understanding	Communicate new mathematical ideas	Explain findings and results from data analysis
Conjecture, generalize, prove	Write formal or informal proofs	Reason inductively or deductively
Solve nonroutine problems and make connections	Recognize, generate, or create patterns	Synthesize content and ideas from several sources

Note.—The full list of activities for each cognitive demand level is available in the SEC online survey.

reporting of more elementary topics (e.g., if a teacher uses addition of fractions as an introduction to a lesson on multiplication of fractions), given the survey scales, the proportion of the total year's instruction and the influence on redundancy in such a case would be small.

All of the results presented here are therefore based on teacher self-report, an important potential limitation of the research that is addressed in more detail in the Discussion section. Nonetheless, investigations of the quality of the SEC teacher survey data indicate that teachers' reports of a single lesson's content in mathematics are moderately correlated with ratings of the same lesson provided by trained observers (Porter, Kirst, Osthoff, Smithson, & Schneider, 1993), and high school mathematics teachers' instructional alignment with a target assessment (based on SEC data) is correlated .45 with student achievement gains on that assessment (Gamoran et al., 1997). Importantly for this analysis, instructional alignment was only predictive of achievement gains when content was defined at the intersection of specific topics and levels of cognitive demand (Gamoran et al., 1997). When content was defined in terms of topics only or cognitive demand levels only, the relationship was not significantly different from zero.

Content analysis. The same content taxonomy is used for teacher surveys as is used for content analyses of standards and assessments. For content analysis of state standards, three to five trained subject-matter experts are involved for each document. The content-analysis procedure is discussed in detail elsewhere (Porter, Polikoff, Zeidner, & Smithson, 2008). Each content analyst examines the content standards at the finest-grained level of detail available (e.g., objectives), with each objective receiving equal weighting.[1] The content in each objective is evenly divided among one to six cells in the SEC taxonomy. Multiple cells are allowed because some objectives cover multiple topics or levels of cognitive demand. For instance, the objective "the student will multiply and divide decimals and one-digit fractions" might be placed in the cells Multiplying Fractions, Dividing Fractions, Multiplying Decimals, and Dividing Decimals, all at the level of Perform Procedures. Then the one point for that objective would be split evenly among the four cells (i.e., each would receive ¼ point). The result of the content analysis of the objectives is a matrix for each rater indicating the proportion of total standards content that falls in each cell in the SEC framework. The matrices are then averaged across raters. Two studies of the reliability of these content-analysis procedures indicate that generalizability coefficients are generally 0.75 or greater for four or more raters (Porter, 2002; Porter et al., 2008).

Sample

Teacher sample. Since 2003, 12,344 teachers from 31 states have completed the SEC in mathematics grades K–8 for various research purposes. These are the teachers from which the sample is drawn. The analytic samples for each grade-to-grade comparison are shown in Table 3. To illustrate the meaning of each grade-to-grade sample, consider the sample in the top row. This sample includes all kindergarten teachers who are in the same school as at least one first-grade teacher, as well as all first-grade teachers who are in the same school as at least one kindergarten teacher. There are a total of 358 such kindergarten teachers and 396 such first-grade teachers. In contrast, there are 433 first-grade teachers who are in the same school as at least

Table 3. Teacher, School, District, and State Sample

Course A Name	Teachers	Course B Name	Teachers	Schools	Districts	States
Kindergarten	358	First grade	396	131	68	17
First grade	433	Second grade	432	167	81	19
Second grade	527	Third grade	606	234	117	19
Third grade	745	Fourth grade	768	293	156	21
Fourth grade	795	Fifth grade	766	342	189	23
Fifth grade	622	Sixth-grade MS math	601	291	194	26
Sixth-grade MS math	676	Seventh-grade MS math	604	306	214	25
Sixth-grade MS math	244	Seventh-grade pre-algebra	115	85	72	16
Seventh-grade MS math	585	Eighth-grade MS math	453	258	189	25
Seventh-grade MS math	435	Eighth-grade pre-algebra	334	181	141	19
Seventh-grade pre-algebra	85	Eighth-grade algebra	84	46	42	14
Total	7,366		7,366	1,049	487	27

Note.—MS = middle school.

one second-grade mathematics teacher in the sample. It is possible for a first-grade teacher to be in both samples—that is, some first-grade teachers in the sample are in schools where kindergarten teachers and second-grade teachers were also surveyed. In all cases, the requirement for inclusion in the analytic file was that the teacher must be in the same school as at least one teacher in the preceding or succeeding grade.[2] In total, there are 7,366 unique teachers in the database, spread across 1,049 schools in 487 districts and 27 states.

Certain descriptive statistics for the teacher sample are compared with national averages obtained from the Digest of Education Statistics (Snyder, Dillow, & Hoffman, 2009) in Table 4. These indicators were selected because data were available in both the SEC and the Digest. As compared to national averages, the sample of teachers used here is almost exactly as educated and slightly more likely to be male. The classes represented here are quite close to national averages in terms of class size but somewhat off in terms of student race. Overall, the teachers and classes in the sample are similar to national averages on the characteristics measured here.

Content standards sample. Also analyzed are content-analysis data for state and Common Core standards, collected since 2002, again based on the SEC

Table 4. Sample Descriptive Statistics

Grade	Teachers	Class Size	% Non-White	Teacher % Male	Teacher % Master's+
K	358	19.63	33.12	10.11	41.60
1	433	19.86	32.84	8.03	45.99
2	527	20.14	36.48	11.59	47.99
3	745	19.87	35.76	10.99	54.72
4	795	20.53	33.42	14.96	51.86
5	622	21.70	33.32	19.28	49.80
6	676	22.77	35.94	23.34	48.23
7	244	22.43	34.17	27.11	46.71
8	585	22.25	33.39	28.52	49.45
Total	7,366	20.36 [a]	34.37	18.73	49.14
National average		20.40 [a]	43.50	16.67	49.20

Note.—Population data are from 2008 Digest of Education Statistics.
[a] Indicates data are for grades K–5 only.

Table 5. State Standards in the Analytic Sample

Standard	Grades	Years in Use
Alabama	6–8	2003–2009
California	K–4	1997–present
Delaware	1–8	2006–present
Idaho	K–8	2006–present
Illinois	1–8	1997–present
Indiana	K–8	2000–present
Kansas	K–8	2003–present
Maine	6–8	2007–2009
Michigan	2–5	2004–present
Minnesota	K–8	2007–present
Montana	3–8	1998–present
North Carolina	3–8	2003–present
New Hampshire	K–8	2006–present
Ohio	K–8	2001–present
Oklahoma	1–8	2003–2009
Oregon	K–8	2002–2008
Vermont	K–8	2004–2010
Common Core	K–8	n/a

procedures. A state's standards were included in the analysis if two or more consecutive grades' standards for that state were in the database. In all cases but one (Oregon, which revised its standards for the 2008–2009 year), the standards in the database are the state's current content standards as of 2009. The states included in the sample with their corresponding grades are listed in Table 5. There are 17 states represented at all in the standards database, and eight complete sets of K–8 standards. Also included in the analysis are the grades K–8 Common Core State Standards in mathematics.

Analysis

The four research questions call for different analytic strategies, each beginning with the teacher-survey and/or content-analysis data. Before beginning any analyses, a small data modification was necessary to account for the fact that the list of topics in the K–8 SEC taxonomy changed slightly in 2006. To address this change, any topic that did not appear in the taxonomies both before and after the switch was deleted and the proportions were renormalized to add to 1. This change could inflate the estimates of redundancy if the deleted topics were less redundant than typical topics, though the average teacher in the database only had 9.6% of his or her instruction removed in the process, so the inflation could be no larger than 9.6%.

Redundancy across grades. The first question asks the extent to which instruction is redundant from grade to grade. Because it is not possible to identify or track students, the conceptual strategy is to identify possible course sequences for students and examine the degree of redundancy and new material students would experience from grade to grade. Within-grade pairs (e.g., K and 1) of teachers within each school are randomly sorted, and instruction for pairs of K–1 teachers is compared. The teachers are compared in two ways. First, a redundancy index for comparing one teacher's instruction to the other is calculated. The index is

$$\text{Proportion redundant content} = \sum_i \min \quad (x_i, y_i),$$

where x_i is the proportion of content in cell i of matrix x (the first-grade teacher's instruction) and y_i is the proportion of content in cell i of matrix y (the kindergarten teacher's instruction). If the instruction is exactly the same between kindergarten and first grade, $x_i = y_i$ for all i. Thus, the index will sum to 1 (because both x_i and y_i individually sum to 1). In contrast, if instruction is completely different between kindergarten and first grade, the index will sum to 0 because either x_i or y_i will equal 0 for all i. Thus, the index ranges from 0 to 1 and indicates the proportion of content in common across grades.

This index takes instruction of the first-grade teacher to be redundant only up to the point at which the proportional coverage of the SEC cell by the first-grade teacher equals the coverage of the kindergarten teacher. For instance, suppose the kindergarten teacher spends 25% of her time on counting, 50% on adding integers, and 25% on subtracting integers. Suppose also that the first-grade teacher spends 25% on adding integers, 50% on subtracting integers, and 25% on multiplying integers. The minima of the four covered cells would be 0% for counting and multiplying integers and 25% for adding and subtracting integers. Thus, the redundancy index would be 0.50 in this case. An alternative way of calculating redundancy might be to sum over all SEC cells for the first-grade teacher that were taught at all by the kindergarten teacher, which in this case would give redundancy of 0.75. The former approach results in more conservative (i.e., lower) estimates of redundancy and is equivalent to the alignment index that showed moderate correlations with value-added to student achievement (Gamoran et al., 1997).

Second, the proportion of the first-grade teacher's instruction that is on SEC cells not taught at all by the kindergarten teacher is calculated, as shown in the equation below, with y_i and x_i having the same meanings as above.

$$\text{Proportion new content} = \sum_{i|x_i=0} y_i.$$

It is important to note that, in the case of spiraled instruction (i.e., returning to previously taught topics but at higher levels of cognitive demand), this content would fall under new content rather than redundant content, because the previous teacher would not have spent any time on that topic at the higher cognitive demand level.

The two indices are calculated for each pair of K–1 teachers, and the teachers are then re-randomized repeatedly until both indices are calculated for every possible within-school combination of K–1 teachers. Repetitions of particular pairs of teachers are deleted, so each teacher pair is represented just once. The result is a distribution of redundancy and new content indices; for each grade pair, the mean and standard deviation of the indices are presented, and a pair of illustrative distributions is presented using histograms.

The effect of middle school course sequencing. To investigate whether redundancy is related to middle school course sequencing, the redundancy and new content indices at the middle school grades are disaggregated based on the type of grade-to-grade transition. There are two possibilities for moving from sixth to seventh grade—moving from middle school mathematics to either middle school mathematics or to pre-algebra. There are three possibilities for moving from seventh to

Table 6. Average Redundancy and Percent New Material from Grade to Grade

Course A Name	Course B Name	Average Redundancy	*SD*	Average New Material	*SD*
Kindergarten	First grade	.38	.12	.48	.19
First grade	Second grade	.42	.12	.42	.19
Second grade	Third grade	.45	.13	.40	.19
Third grade	Fourth grade	.49	.12	.32	.19
Fourth grade	Fifth grade	.49	.13	.31	.20
Fifth grade	Sixth-grade MS math	.50	.15	.27	.21
Sixth-grade MS math	Seventh-grade MS math	.48	.15	.30	.20
Sixth-grade MS math	Seventh-grade pre-algebra	.48	.14	.30	.20
Seventh-grade MS math	Eighth-grade MS math	.46	.15	.31	.22
Seventh-grade MS math	Eighth-grade pre-algebra	.49	.15	.29	.21
Seventh-grade pre-algebra	Eighth-grade algebra	.38	.15	.33	.22

Note.—MS = middle school.

eighth grade—moving from middle school mathematics to either middle school mathematics or pre-algebra, or moving from pre-algebra to algebra. The means of the two indices are compared for these different kinds of transitions.

Redundant and new topics. For each teacher, the proportions in their instructional matrix are transformed to binary indicators indicating whether or not they covered each SEC cell. Then the proportions of teachers at each grade covering each SEC cell are calculated and the teacher proportions are compared for adjacent grades. Redundant content is represented by the SEC cells that are covered by the highest proportions of teachers in both grades. New content is represented by the SEC cells with the greatest increase in proportion of teachers covering that content from grade to grade. For the sake of conciseness, the top three cells on each index for each grade-to-grade transition are presented, though it would be possible to rank all 915 cells on the two indices. Patterns in the full distributions of the two indices are also discussed.

The redundancy of state and Common Core standards. For each available set of content-analyzed documents in the database, the same redundancy and new content indices are created and averages are taken across grades. The indices are also calculated and presented for the new Common Core State Standards. To the extent that there are teachers in the sample in the states with content-analyzed standards and assessments, correlations of indices with state standards indices are examined. The hypothesis is that teachers in states with more redundant standards would be more likely to practice instruction that is redundant.

Results

The Redundancy of Instruction across Grades

The first question asks how redundant instruction is within schools across grades; the results are presented in Table 6. The average redundancy of a first-grade teacher's instruction with a kindergarten teacher's within schools is 0.38. This means that 38% of the content covered by a typical first-grade teacher is on SEC cells that are also covered by the typical kindergarten teacher from the same school, and in the same proportions. The redundancy of instruction from grade to grade increases mono-

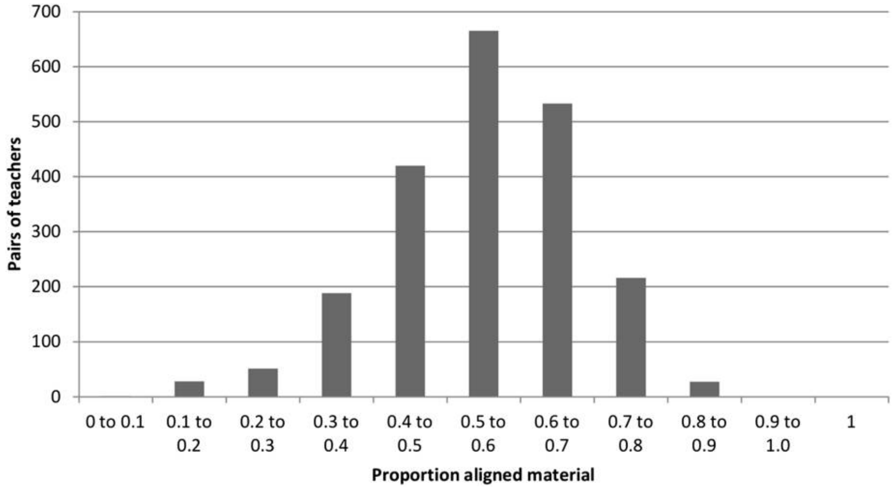

Figure 1. Distribution of the proportion of redundant material in third-grade mathematics classes as compared to second-grade mathematics classes.

tonically from first grade through sixth grade, where the average value is 0.50. There is a small decrease for the middle school transitions. In all cases, however, the average redundancy is over one-third. Translated to the metric of a 180-day school year, between 68 and 90 days' instruction, depending on grade, is on SEC cells already covered in the same proportions in the previous grade.

To get a better sense of the distribution of redundancy across teachers, Figure 1 displays a histogram of all redundancy indices for teacher pairs in grades 2 and 3. The mean for this grade is 0.45 with a standard deviation of 0.13. Examination of the distribution reveals a slightly nonnormal, negatively skewed distribution, with a modal value between 0.5 and 0.6. Almost all teachers have instruction that is at least moderately redundant with the instruction offered in the previous grade, with just 4% of teacher pairs having redundancy indices below 0.3. More than 68% of all teacher pairs indicate third-grade teachers spending at least half of their instructional time covering content that was already taught in the same proportions in second grade.

A second category of instructional content is new material—instruction on SEC cells that were not at all covered at the previous grade. The right side of Table 6 displays the average values for the proportion of new content at each grade as compared to the previous grade. The grade with the most new content is first-grade mathematics, where 48% of the content is new as compared to the kindergarten mathematics classes in the same school. In second grade, 42% of instructional content is new, the same proportion as is redundant with first-grade instruction. For all other grades, the proportion of new content is smaller than the proportion of redundant content. Again, the grade where the instruction is the least new (i.e., most redundant) is sixth grade, where just over one-quarter of the content is on SEC cells that were not covered in fifth grade. Across all grades except K–3, one-third or less of the total content in any given year is new as compared with the content from the previous year. In a 180-day school year, the average number of days spent on new material is between 48 and 86 days, depending on grade.

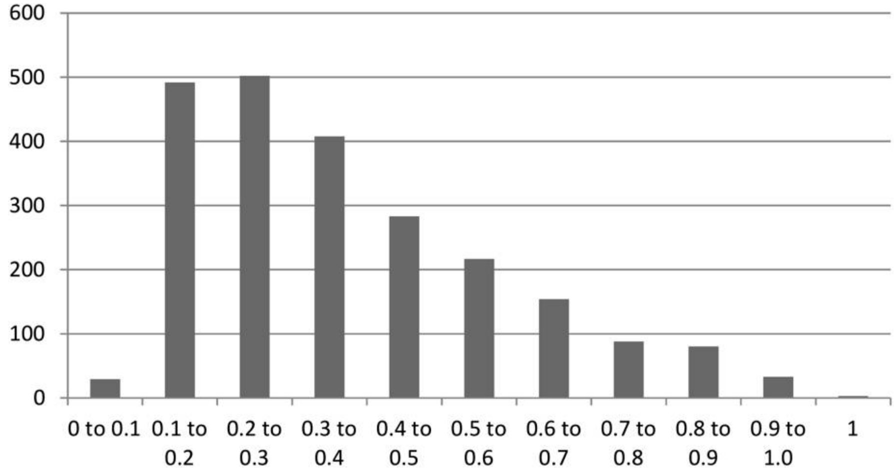

Figure 2. Distribution of the proportion of new material in sixth-grade mathematics classes as compared to fifth-grade mathematics classes.

Figure 2 shows a histogram for the proportion of new material in teachers' instruction in sixth grade as compared to fifth grade. Recall that this was the grade at which there was the least new material—just 27% on average, with a standard deviation of 0.21. Examination of the histogram reveals that the distribution is positively skewed. Approximately 43% of teachers spend between 10% and 30% of their time on content that was not taught in the previous grade, with an additional 30% spending between 30% and 50% of instructional time on new content. Just one-quarter spend more than half the year's time teaching content that was not taught in the previous grade. Clearly, most teachers are spending no more than a moderate proportion of total instructional time introducing new material in sixth-grade mathematics.

The remaining proportion of instructional time—the proportion that is neither completely redundant with the previous grade's instruction nor on SEC cells that were not at all covered in the previous grade—is instruction on SEC cells that were covered in the previous grade, but where the emphasis in the current grade is larger than the emphasis in the previous grade. For example, if a teacher devoted 3% of her time to multiplying fractions at the level of Perform Procedures in fifth grade, and the next teacher devoted 7% in sixth grade, there is a 4% extra emphasis on that SEC cell in sixth-grade instruction. While this was not included in the main redundancy index, as mentioned previously, this surplus emphasis could also be considered a type of redundancy, insofar as the instruction is on topics that were taught at the previous grade. The proportion of instruction that represents a surplus coverage of already taught material ranges from 14% in kindergarten to 28% in eighth grade, with a nearly monotonic increase. When added to the redundancy indices, these figures indicate that 60% to 70% of instructional time across grades is spent on topics taught at previous grades.

Redundancy across Middle School Tracks

It might be hypothesized that more advanced students—those who enroll in pre-algebra in seventh grade and algebra in eighth grade—are exposed to more new content than students who are in the average or remedial tracks in mathematics. To investigate this question, the redundancy and new content indices for the middle school grades were disaggregated by academic track. The results are presented at the bottom of Table 6.

For the transition from sixth to seventh grade, there was no difference in redundancy between teachers who said they taught seventh-grade pre-algebra versus seventh-grade middle school mathematics. Indeed, the average indices for both redundancy (0.48) and new material (0.30) are identical regardless of track placement. It could be possible that the seventh-grade pre-algebra class is more advanced in some way than the seventh-grade middle school mathematics class, but the evidence suggests that the amount of instructional time devoted to new material in seventh grade is unrelated to whether or not the student is preparing to take algebra in eighth grade.

For the transition from seventh to eighth grade, there are three possibilities. If students took middle school mathematics in seventh grade, they could take middle school mathematics again in eighth grade (the lowest track) or pre-algebra (the middle track). In contrast, students could have taken pre-algebra in seventh grade, in which case they would take algebra in eighth grade (the highest track). The bottom three rows of Table 6 summarize the redundancy in these three cases. If the student is coming from middle school mathematics, the redundancy of the next course is similar regardless of whether it is more middle school mathematics or pre-algebra; there is slightly more redundancy and less new content in eighth-grade pre-algebra. In contrast, there is slightly less redundancy of eighth-grade algebra instruction with seventh-grade pre-algebra, with only 38% redundant content. Overall, however, the results suggest that, regardless of middle school track, there is a moderate amount of redundancy that does not vary substantially across track.

Redundant and New Topics at Each Grade

The third research question asks which content is the most redundant and most new from grade to grade. The most redundant SEC cells—those taught by large proportions of teachers across grades—are presented in Table 7. At seventh and eighth grades, the tracks are aggregated rather than presented separately. Three SEC cells per grade comparison are presented. The first commonality among the redundant cells is that all but two of the 24 are at level C, Perform Procedures. Procedural skills are emphasized repeatedly across grades more than other more conceptual skills. A second commonality is that certain topics appear as the most redundant across multiple grades. For instance, procedures for adding and subtracting whole numbers and integers is one of the most redundant topics for grades K–1, 1–2, 2–3, and 4–5. Across all of the grades, the most redundant topics are in areas of number sense.

In addition to those SEC cells identified in Table 7, there are dozens of other SEC cells that are covered by large proportions of teachers in each grade pair. In grades K–1, there are 12 cells that are covered by 80% or more of teachers in both grades. There are 27 such cells in grades 1–2, 36 in grades 2–3, 58 in grades 3–4, 65 in grades 4–5, 81 in grades 5–6, 80 in grades 6–7, and 79 in grades 7–8. At all grades, the majority of redundant SEC cells are at levels B and C in the SEC taxonomy—memorization and performing procedures, respectively. For instance, of the 65 cells that are emphasized by 80% or more of both fourth- and fifth-grade teachers, 19 are at

Table 7. Most Redundant Content for Each Grade Pair

Topic	Cognitive Demand Level	Proportion Covering in Previous Grade	Proportion Covering in Current Grade
Grade 1:			
Add/subtract whole numbers and integers	C	.91	.94
Use of measuring instruments	C	.89	.91
Number sense: patterns	C	.88	.90
Grade 2:			
Add/subtract whole numbers and integers	C	.94	.95
Number sense: place value	C	.92	.96
Number sense: operations	C	.92	.94
Grade 3:			
Number sense: place value	C	.96	.97
Add/subtract whole numbers and integers	C	.95	.95
Number sense: place value	D	.95	.95
Grade 4:			
Number sense: place value	C	.97	.96
Number sense: operations	C	.96	.96
Number sense: place value	D	.95	.95
Grade 5:			
Number sense: operations	C	.95	.94
Number sense: place value	C	.94	.93
Add/subtract whole numbers and integers	C	.93	.95
Grade 6:			
Number sense: fractions	C	.94	.94
Number sense: decimals	C	.93	.94
Equivalent and nonequivalent fractions	C	.94	.92
Grade 7:			
Number sense: fractions	C	.96	.92
Number sense: percents	C	.92	.92
Number sense: operations	C	.93	.91
Grade 8:			
Number sense: percents	C	.93	.93
Number sense: fractions	C	.92	.92
Number sense: ratios and proportions	C	.91	.91

level B and 25 are at level C. Just 18 are at level D, one is at level E, and none is at level F. This is evidence that the redundant material across grades is disproportionately focused on procedures or other rote skills, rather than more problem-based or conceptual skills.

Table 8 focuses on the content that is most new in each grade. These are the SEC cells that show the biggest grade-to-grade increase in the proportion of teachers covering them. Of note in Table 8 is that, at each grade, at least two of the top three new SEC cells come from the same topic. These topics are metric system in first grade, multiply whole numbers and integers in second grade, angles in third grade, multiply decimals in fourth grade, multiply fractions in fifth grade, divide fractions in sixth grade, absolute value in seventh grade, and rate of change/slope/line in eighth grade. As with the most redundant content, the major new content tends to come from the lower two levels of cognitive demand—16 of the 24 SEC cells in Table 8 are from levels B or C. However, there are also some new content cells that come from level D, the middle level of cognitive demand.

Another important trend revealed in Table 8 is that the new content in the earlier grades is more likely to be true new content, in the sense of being SEC cells that are not covered by many teachers in previous grades. In contrast, in later grades, the new

Table 8. Content with the Largest Grade-to-Grade Increase in Proportion of Teachers Covering

Topic	Cognitive Demand Level	Proportion Covering in Previous Grade	Proportion Covering in Current Grade	Increase
Grade 1:				
Metric system	C	.06	.48	.42
Odd/even/prime/composite/square numbers	C	.35	.75	.40
Metric system	B	.06	.43	.37
Grade 2:				
Multiply whole numbers and integers	B	.13	.63	.49
Multiply whole numbers and integers	C	.15	.63	.48
Multiply whole numbers and integers	D	.13	.57	.44
Grade 3:				
Divide whole numbers and integers	D	.38	.81	.43
Angles	C	.33	.76	.43
Angles	D	.28	.71	.42
Grade 4:				
Multiply decimals	C	.14	.53	.39
Multiply decimals	B	.15	.51	.36
Multiply decimals	D	.12	.46	.34
Grade 5:				
Multiply fractions	C	.34	.74	.39
Multiply fractions	B	.34	.71	.37
Divide fractions	C	.29	.66	.37
Grade 6:				
Divide fractions	C	.67	.88	.21
Absolute value	C	.39	.57	.18
Divide fractions	D	.63	.80	.17
Grade 7:				
Absolute value	D	.41	.76	.35
Absolute value	C	.45	.80	.35
Linear and nonlinear relations	D	.23	.57	.34
Grade 8:				
Rate of change/slope/line	C	.40	.78	.39
Rate of change/slope/line	D	.37	.75	.37
Rate of change/slope/line	B	.37	.72	.34

content is mainly SEC cells that have gone from moderate emphasis (e.g., 40% to 60% of teachers covering) to high emphasis (70% to 90% of teachers covering). As with the data on redundancy, this indicates a slowing down in the accumulation of new content as grades progress. A final trend to note is that, in grades 5–7, two topics appear as major new topics in multiple grades. These are dividing fractions, which appears in grades 5 and 6, and absolute value, which appears in grades 6 and 7. That a particular SEC cell appears as major new content in consecutive grades is perhaps indicative of disagreement about the proper timing for introduction of these topics.

Looking across the full set of SEC cells at each grade-to-grade transition, two patterns become apparent. The first pattern is that, while many topics are added to the curriculum, especially in early grades, topics leave the curriculum at a much slower rate. For instance, in the transition from kindergarten to first grade, there are a total of 490 SEC cells that see an increase in teacher coverage (a greater proportion of teachers covering them in first grade as compared to kindergarten). In contrast, there are just 12 SEC cells that see a decrease in coverage at the same grades. In second grade, the numbers are 342 and 160, respectively. At all grade-to-grade transitions there are more topics added to the curriculum than subtracted from the curriculum.

Table 9. Average Redundancy and Percent New Content in Content Standards from Grade to Grade

	All States		States with K–8 Standards		Common Core Standards	
Grades	Redundancy	New Content	Redundancy	New Content	Redundancy	New Content
K & 1	.47	.38	.50	.36	.41	.39
1 & 2	.53	.29	.53	.28	.60	.19
2 & 3	.46	.34	.46	.33	.37	.35
3 & 4	.40	.37	.33	.41	.35	.50
4 & 5	.45	.32	.42	.35	.41	.34
5 & 6	.36	.42	.39	.41	.22	.71
6 & 7	.37	.43	.36	.41	.35	.54
7 & 8	.36	.42	.39	.40	.17	.70

The second apparent pattern is that there is far from consistent agreement about the topics to be introduced at particular grades, especially in later elementary and middle school. Not one SEC cell at any grade transition shows an increase of 50% or greater in the proportion of teachers covering it. Furthermore, the number of SEC cells showing even a moderate increase in proportion of teachers covering them (arbitrarily defined as a 25% or more increase) diminishes substantially across grades, from 50 between second and third grades to just 10 from seventh to eighth. In other words, not only is there apparent disagreement about the timing of proper introduction of key topics, but this disagreement tends to increase at higher grades.

The Redundancy of State and Common Core Standards

The final research question asks about the redundancy of state and the Common Core mathematics content standards, as a first attempt at understanding the extent to which these standards have been and could be influences on the redundancy of instruction. Table 9 compares standards grade to grade within state, presenting the average redundancy and new content indices for standards at each grade-to-grade comparison. The left side of the table shows the results when all available states are included, and the right side shows the results for only those states with complete sets of K–8 standards in the database. The redundancy of standards across grades is highest in first to second grade at 53%, and generally lowest in fifth to eighth grade at 36% to 39%. The results are similar whether the total set of 17 states or the set of eight complete states are analyzed. In terms of magnitude, the redundancy of state standards is similar to the redundancy of instruction, with values generally in the range of 0.36 to 0.50. The least new content is found in standards in the early grades, K–3. The standards for grades 5–8 have about 40% new content per year, a somewhat higher proportion of new content than is found in the enacted curriculum.

Also provided in Table 9 are the same indices for the newly developed Common Core State Standards in mathematics, soon to be in use in over 40 states. The results here are similar for elementary school, with roughly 40% redundancy and 35% new content at each grade. In middle school, however, the Common Core State Standards appear to be somewhat less redundant than the typical state standards, especially for the grade 5–6 and 7–8 transitions. At those transitions, there is roughly 20% redundancy and 70% new content. In short, analysis of the Common Core State Standards suggests that, with the exception of two grades in middle school, they are no less redundant than the typical state standards they replaced, despite the stated goal of the developers for an aligned K–12 college- and career-ready system.

An important policy question is whether the redundancy of instruction can be influenced by the redundancy of the target standards. There are 1,913 teachers in the database in states and grades for which there are also content-analyzed standards that were in place at the time they completed the SEC. Teacher-redundancy indices were correlated with state-standards redundancy indices using Pearson's r, with a resulting correlation of -0.005. The correlation for the new content indices is 0.019. Neither of these correlations is statistically different from 0. Examination of the intraclass correlations for redundancy and proportion of new content across states suggests that the lack of correlation is reflective of the fact that the vast majority of the variance in redundancy (greater than 94%) is found within rather than between states. Simply put, though state standards appear to be just as redundant with one another as teachers' instruction is, there is no apparent relationship between the redundancy of content standards and the redundancy of instruction.

Discussion

The content to which students are exposed is a primary influence on the knowledge they will gain in schools. This is a key principle underlying the standards-based reform movement of the last twenty-plus years. Noting the poor performance of U.S. students on international comparisons in mathematics, some have blamed the U.S. mathematics curriculum, arguing that it is overly shallow and redundant (Schmidt et al., 1997, 2001, 2005). A number of studies, mainly predating NCLB, have examined the intended and enacted curriculum in U.S. mathematics classes and found high degrees of repetitiveness in standards and textbooks (Porter, 1989; Schmidt et al., 1997, 2001, 2005).

To investigate whether redundancy remains a feature of the intended and enacted curriculum after early implementation of NCLB, survey data on more than 7,000 teachers' K–8 instruction in the years 2003–2009 from the Surveys of Enacted Curriculum were used. The results indicated a high degree of redundancy from year to year, with 60%–70% of content in most grades redundant with the previous grade's instruction. Redundancy was especially high in late elementary and middle school, and lower in the early primary grades. Redundancy was not substantially different across academic tracks. Certain core content was identified as the most redundant across grades, and this content was mainly procedural in nature. While there was new content in each year's instruction, teachers clearly added topics to their instruction continually across the nine grades, another indication that the middle school curriculum is particularly broad and shallow. Finally, state standards and the Common Core Standards were found to be just as redundant as teachers' instruction, with just 30%–40% of content in a particular state's standards being new from year to year. There was no apparent relationship between the redundancy of state standards and the redundancy of instruction, however, suggesting that merely improving the coherence of state standards might not result in reduced redundancy.

There are several important limitations to these conclusions. The first potential limitation is the quality of the instructional survey data. While there is literature on the quality of teachers' survey responses about their instruction, nearly all of the literature is more focused on instructional practices than instructional content

(Hiebert & Stigler, 2000; Hook & Rosenshine, 1979; Mayer, 1999; Ross, McDougall, Hogaboam-Gray, & LeSage, 2003; Wubbels, Brekelmans, & Hooymayers, 1992). This work suggests that the reliability of individual indicators (e.g., individual instructional strategies) is much lower than that of aggregate measures (Mayer, 1999), and that teachers may do an especially poor job of reporting their practice of behaviors that are desirable from the standpoint of representing good teaching under the expectations of current mathematics education reform (Hiebert & Stigler, 2000; Ross et al., 2003). In the case of the SEC, perhaps this would manifest itself in an inaccurate reporting of cognitive demand coverage, such as an overestimation of the coverage of higher levels of cognitive demand. However, the validity studies of the SEC described earlier do indicate moderate correlations among daily logs, SEC surveys, and student achievement gains, even using taxonomies that included more cognitive demand levels (six to nine) than are used here (Gamoran et al., 1997; Porter et al., 1993). Thus, there should be some confidence about the validity and reliability of these data, but it is nonetheless possible that teachers might systematically under- or overstate their topic or cognitive demand coverage, which could result in inflated estimates of redundancy.

The second potential limitation has to do with the sample's representativeness and generalizability. Certainly it cannot be claimed that the results presented here are generalizable to all U.S. teachers. The sample is a convenience sample, albeit a large one drawn from many states and districts and roughly similar to the population of U.S. teachers on certain descriptive variables. The teacher samples are much larger than most previous studies on instructional redundancy, however, and there is no indication that redundant instruction is idiosyncratic to particular schools or teachers. Quite the contrary, there is remarkable consistency across teachers and state standards in the magnitude of instructional redundancy. Nevertheless, the results would have better external validity if the participating teachers had been selected to be representative, and a nationally representative study of instruction using a more detailed instructional measure such as the SEC would represent a substantial contribution to the field.

Finally, this study, like all previous studies of the redundancy of the curriculum, is limited by the framework used to analyze the standards and instruction. While the SEC includes many more topics than frameworks used in prior studies, there are surely topics that some math teachers teach that are not included in the SEC. Also, as noted earlier, the SEC does not differentiate based on complexity in mathematical topics. Both of these weaknesses of the SEC would tend to result in an overestimation of redundancy, though the magnitude of this overestimation is not clear. It would require building consensus about the key content distinctions that matter and utilizing a substantially more finely grained framework to fully account for all the content distinctions that are important to perfectly characterize redundancy.

With these limitations in mind, the results presented here point to a continued, troubling fact about U.S. mathematics instruction: despite more than 2 decades of curriculum reform, instruction remains poorly structured and highly repetitive. Students are exposed to the same content grade after grade, and there appears to be little consistency across teachers, districts, and states as to the proper timing for introduction of skills and techniques. At first glance, curriculum standards seem unlikely to help improve the situation much, as this analysis suggests they are unrelated to the redundancy of the enacted curriculum. However, one possible hypothesis for the apparent lack of relation between standards and instruction in terms of redundancy is that the standards are poorly structured and sequenced. That is, while the original theory of change for standards-based reform specified that instructional coherence was paramount, the current set of state standards have not achieved this goal. Future research should probe this hypothesis, focusing on the instructional coherence in sites where the instructional guidance system is more carefully designed to maximize coherence.

Another potential hypothesis for the high overall redundancy that merits exploration is the content of textbooks. It is widely known that textbook companies, responding to the 50 sets of fairly divergent state standards under NCLB (Porter et al., 2009), produced textbooks that were encyclopedic in content coverage—touching on every topic that was covered in any state's standards. As the Common Core Standards roll out and are adopted in the majority of U.S. schools, it will be important to examine the extent to which publishers narrow the content focus of their new textbooks. Even though the findings presented here do not indicate that the Common Core Standards are appreciably less redundant than typical state standards in elementary school, their adoption may still result in curriculum and textbook improvements because textbooks will no longer have to be responsive to 50 different sets of standards.

While the results of this work are largely in line with research from TIMSS (Schmidt et al., 1997, 2001, 2005), it would be a worthwhile endeavor to utilize a more comprehensive framework such as the SEC in investigating content coverage internationally. The work of Schmidt and his colleagues, along with U.S.–specific work on opportunity to learn, indicates that differences in content coverage across classrooms are a potentially critical contributor to poor U.S. mathematics student achievement (Gamoran et al., 1997; Schmidt, Cogan, Houang, & McKnight, 2011; Schmidt et al., 2001). Particularly important in the TIMSS work was developing an understanding of the progression of mathematics skills across grades in high-performing countries (Schmidt et al., 2005). The methods used here could contribute to deepening that understanding. Developing a more micro-level and international understanding of how standards, curriculum materials, and teachers' instruction relate to student learning would undoubtedly help the mathematics curriculum improvement efforts taking place in the United States.

Another worthwhile investigation would be examining the extent to which redundancy varies across settings. While the means for the indices studied here indicated high degrees of redundancy, there were at least some teachers at all grades who practiced less redundant instruction. It would be useful to identify who these teachers are, what kinds of students they teach, and what kinds of schools they teach in. Perhaps an even more important topic is understanding how these teachers are able to teach less redundant content than their peers—what are the curriculum sources, professional communities, and professional development activities that enable teachers to teach better structured, less redundant curricula?

While many mathematics education experts believe the Common Core Standards are an improvement over most state standards (e.g., Cobb & Jackson, 2011), the results of this work suggest that more supports will need to be offered to teachers in order for the Common Core to have maximum impact. These supports were laid out in the original vision of standards-based reform (e.g., Smith & O'Day, 1991); they include (*a*) professional development for teachers to help them implement the standards as intended, (*b*) improved alignment of curriculum materials with standards, and (*c*) improved alignment of assessments with standards. These supports will be especially important for experienced mathematics teachers, many of whom will now have taught three or more sets of standards, and for elementary teachers, many of whom are not mathematics experts. Even with these supports, if mathematics teachers remain far more willing to add topics to their curriculum than to remove them (Floden, Porter, Schmidt, Freeman, & Schwille, 1980), it will be difficult to see dramatic improvements in redundancy. In short, the strategy must be to first identify the goal for improving instruction and then align the standards with other policy instruments in the direction of achieving that goal. If the system is not structured in this way, decades of educational research suggest that the impact on instruction and, therefore, achievement, will be less than is desired.

Notes

Thank you to John Smithson for providing the data for this analysis.

1. While it is possible that some objectives are more "important" than other objectives, it would be difficult to account for importance in content analysis. Establishing importance would require an a priori importance system that had common interpretations across content analysts and also matched the importance weightings assigned by the users of standards (i.e., teachers). Weighting objectives evenly is therefore the most replicable and defensible approach.

2. Year of SEC administration was ignored for the purposes of these analyses, though 87.9% of teacher pairs analyzed were from the same or immediately adjacent years. Furthermore, all results presented here were statistically identical to results obtained when the teacher sample included only teacher pairs from the same or immediately adjacent years.

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