AME 526 – Partial Differential Equations for Engineering Applications Spring 2025

Unit: 4

Time: Mon & Wed 10:00 am – 11:50 am **Location**: RTH 105

Instructor: Rui Xu Office: OHE 412A

Office hours: Mon & Wed 1-2 pm in OHE 412A, additional times by appointment

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Course Description

This course introduces the concept and analytical approaches to solving partial differential equations for graduate students. The lectures are structured with necessary background materials leading to analytically solving the partial differential equations. The first half of the course starts with solution techniques for linear ordinary differential equations, followed by the power series method, special functions (e.g., Bessel functions and Legendre polynomials), Fourier series and integrals, and Sturm-Liouville theory. The first-half materials closely align with the solution techniques for partial differential equations. The second half of the course primarily focuses on the analytical solutions to partial differential equations, and we will cover the analytical techniques in cartesian, cylindrical, and spherical coordinates, along with examples in heat and mass transfer, and fluid mechanics.

Learning objectives

This course intends to build foundations for graduate students in partial differential equations, along with the analytical solution approaches. Upon completing this course, students are expected to achieve the following goals.

- 1) Analytically solving linear homogeneous ordinary differential equations (ODEs) and also solving non-homogeneous ODEs using the method of undetermined coefficients and the variation of parameters.
- 2) Analytically solving non-constant coefficient ODEs with the power series method.
- 3) Understanding special functions such as Bessel functions and Legendre polynomials.
- 4) Performing Fourier-series expansion to even and odd functions using sine and cosine series with $n\pi$ and an arbitrary period.
- 5) Performing double variables Fourier series and integral expansions.
- 6) Performing expansions of special functions such as Bessel functions and Legendre polynomials.
- 7) Understand Sturm-Liouville theory and orthogonal eigenfunctions.
- 8) Understanding the concept of partial differential equations (PDEs) and the classification.
- 9) Analytically solving PDEs (Ellipse, parabolic, and hyperbolic) in cartesian coordinates, using methods such as separation of variables and eigenfunction expansions.
- 10) Analytically solving PDEs in cylindrical coordinates and spherical coordinates.
- 11) Establishing physical intuition of differential equations to real engineering physics problems.

Prerequisite(s): N/A

Recommended preparation: Basic calculus and linear algebra.

Course Notes

The course uses DEN D2L online system (https://courses.uscden.net/d2l/login). All course materials, including lecture notes, lecture videos, syllabus, homework and solutions, and any supplementary materials will be posted online in the DEN D2L website.

Required Textbook

Satwindar S. Sadhal, "Fourier Analysis, Eigenfunction Expansions, and Partial Differential Equations," Mathematical Education for Engineering. ISBN-13: 978-0991368310.

Amazon link: https://www.amazon.com/dp/0991368312?ref =pe 93986420 77495752.

Optional reading Textbook

Peter V. O'Neil, "Advanced Engineering Mathematics," Cengage Learning. ISBN-13: 978-1305635159.

Grading Breakdown

Homework 30% Midterm exam 30% Final exam 40%

Assignment Submission Policy

- Homework assignments are due on **each Friday at 12 pm (noon)**, starting from Week 3 on Friday Jan. 31 (see the detailed course schedule next page).
- Each homework assignment should be **submitted electronically as a single PDF file** via the DEN D2L website (https://courses.uscden.net/d2l/login). If you have a paper-based version of your homework assignment, you can use a scanner or any existing smartphone apps that use the phone camera as a scanner. Please make sure to append all pages into a single PDF document before submitting.
- Both hand-written and typed solutions are acceptable. Please provide organized solutions that include all necessary assumptions (if any) and steps.
- Late homework submission will be marked down by a 25% deduction in the grade every 24 hours late. No late submissions are accepted beyond 3 days (72 hours) of the due date, which is the following Monday at 12 pm (after the previous Friday due date). Late submissions could be occasionally excused due to emergencies, by emailing the instructor as soon as possible and discussing alternative arrangements.

Grading Timeline

Graded homework assignments, exams, and numerical grades will be posted online within 2 weeks after the submission deadline.

Exams

Exams will be open book and notes. There will be two exams, and the schedules are:

- Midterm exam: Wednesday, March 5, 10-11:50 am, RTH 105.
- Final exam: Monday, May 12, 8-10 am, RTH 105.

Course Schedule

Week	Date	Lecture	Topic	Homework
1	01/13	1	Intro to ordinary differential equations (ODEs),	
			solution of homogeneous ODEs	
	01/15	2	Solution of non-homogeneous ODEs	
2	01/20	_	Martin Luther King Day, no lecture	HW1 online
	01/22	3	Variation of parameters, the Euler equation	
3	01/27	4	Intro to power series solutions	HW2 online
	01/29	5	Power series solutions with logarithmic, general	HW1 due 01/31
			procedure for second-order equations	
4	02/03	6	Bessel functions, Legendre Polynomials	HW3 online
	02/05	7	Intro to Fourier series, even and odd functions	HW2 due 02/07
5	02/10	8	Fourier series of non-symmetric functions	HW4 online
	02/12	9	Double Fourier series, infinite-domain expansions	HW3 due 02/14
6	02/17	_	Presidents' Day, no lecture	HW5 online
	02/19	10	Intro to partial differential equations (PDEs),	HW4 due 02/21
			classification, initial and boundary conditions	
7	02/24	11	Ellipse (Laplace's) equation, Separation of variables,	HW6 online
			eigenfunction expansion	HW5 due 02/28
	02/26	12	Superposition principle, parabolic equation	
8	03/03	13	Hyperbolic (wave) equation, infinite domain	
	03/05	_	Midterm exam (10-11:50 am, RTH 105)	
9	03/10	14	Mixed boundary condition, three-variable problem	HW7 online
	03/12	15	Method of Characteristics, D'Alembert's solution to	HW6 due 03/14
			wave equation, geometrical limitations	
			03/16 – 03/23 Spring recess	
10	03/24	16	PDEs in cylindrical coordinates, eigenfunction	HW8 online
			expansions in θ	HW7 due 03/28
	03/26	17	Eigenfunction expansions in r, Sturm-Liouville	
			theory, orthogonal eigenfunctions	
11	03/31	18	Sturm-Liouville theory, classification of boundary	HW9 online
			conditions for orthogonality	HW8 due 04/04
	04/02	19	Time-dependent PDE, Eigenfunction expansions in r	
			and θ	
12	04/07	20	Time-dependent PDE with non-homogeneous	HW10 online
			boundary conditions	HW9 due 04/11
	04/09	21	PDEs in spherical coordinates, eigenfunction	
			expansions in θ	
13	04/14	22	Hemispheric problems, eigenfunction expansions in r	HW11 online
	0.1.11.6		(time-dependent PDEs)	HW10 due 04/18
4.4	04/16	23	Eigenfunction expansions in r and θ	T 111/14 0 "
14	04/21	24	Classical examples of PDEs	HW12 online
	04/23	25	Laplace transform	HW11 due 04/25
15	04/28	26	Laplace transform	HW12 due 05/02
	04/30	27	Review session	
Final	05/12		Final exam (Monday, May 12, 8-10 am, RTH 105)	

Statement on University Academic Conduct and Support Systems

Academic Integrity

The University of Southern California is foremost a learning community committed to fostering successful scholars and researchers dedicated to the pursuit of knowledge and the transmission of ideas. Academic misconduct is in contrast to the university's mission to educate students through a broad array of first-rank academic, professional, and extracurricular programs and includes any act of dishonesty in the submission of academic work (either in draft or final form).

This course will follow the expectations for academic integrity as stated in the <u>USC Student Handbook</u>. All students are expected to submit assignments that are original work and prepared specifically for the course/section in this academic term. You may not submit work written by others or "recycle" work prepared for other courses without obtaining written permission from the instructor(s). Students suspected of engaging in academic misconduct will be reported to the Office of Academic Integrity.

Other violations of academic misconduct include, but are not limited to, cheating, plagiarism, fabrication (e.g., falsifying data), knowingly assisting others in acts of academic dishonesty, and any act that gains or is intended to gain an unfair academic advantage.

Academic dishonesty has a far-reaching impact and is considered a serious offense against the university. Violations will result in a grade penalty, such as a failing grade on the assignment or in the course, and disciplinary action from the university itself, such as suspension or even expulsion.

For more information about academic integrity see the <u>student handbook</u> or the <u>Office of Academic Integrity's website</u>, and university policies on <u>Research and Scholarship Misconduct</u>.

Please ask your instructor if you are unsure what constitutes unauthorized assistance on an exam or assignment or what information requires citation and/or attribution.

Students and Disability Accommodations

USC welcomes students with disabilities into all of the University's educational programs. The Office of Student Accessibility Services (OSAS) is responsible for the determination of appropriate accommodations for students who encounter disability-related barriers. Once a student has completed the OSAS process (registration, initial appointment, and submitted documentation) and accommodations are determined to be reasonable and appropriate, a Letter of Accommodation (LOA) will be available to generate for each course. The LOA must be given to each course instructor by the student and followed up with a discussion. This should be done as early in the semester as possible as accommodations are not retroactive. More information can be found at osas.usc.edu. You may contact OSAS at (213) 740-0776 or via email at osasfrontdesk@usc.edu.

Student Financial Aid and Satisfactory Academic Progress

To be eligible for certain kinds of financial aid, students are required to maintain Satisfactory Academic Progress (SAP) toward their degree objectives. Visit the <u>Financial Aid Office webpage</u> for <u>undergraduate</u> and <u>graduate-level</u> SAP eligibility requirements and the appeals process.

Support Systems:

Counseling and Mental Health - (213) 740-9355 - 24/7 on call

Free and confidential mental health treatment for students, including short-term psychotherapy, group counseling, stress fitness workshops, and crisis intervention.

988 Suicide and Crisis Lifeline - 988 for both calls and text messages — 24/7 on call

The 988 Suicide and Crisis Lifeline (formerly known as the National Suicide Prevention Lifeline) provides free and confidential emotional support to people in suicidal crisis or emotional distress 24 hours a day, 7 days a week, across the United States. The Lifeline consists of a national network of over 200 local crisis centers, combining custom local care and resources with national standards and best practices. The new, shorter phone number makes it easier for people to remember and access mental health crisis services (though the previous 1 (800) 273-8255 number will continue to function indefinitely) and represents a continued commitment to those in crisis.

Relationship and Sexual Violence Prevention Services (RSVP) - (213) 740-9355 (WELL) - 24/7 on call

Free and confidential therapy services, workshops, and training for situations related to gender- and power-based harm (including sexual assault, intimate partner violence, and stalking).

Office for Equity, Equal Opportunity, and Title IX (EEO-TIX) - (213) 740-5086

Information about how to get help or help someone affected by harassment or discrimination, rights of protected classes, reporting options, and additional resources for students, faculty, staff, visitors, and applicants.

Reporting Incidents of Bias or Harassment - (213) 740-2500

Avenue to report incidents of bias, hate crimes, and microaggressions to the Office for Equity, Equal Opportunity, and Title for appropriate investigation, supportive measures, and response.

The Office of Student Accessibility Services (OSAS) - (213) 740-0776

OSAS ensures equal access for students with disabilities through providing academic accommodations and auxiliary aids in accordance with federal laws and university policy.

USC Campus Support and Intervention - (213) 740-0411

Assists students and families in resolving complex personal, financial, and academic issues adversely affecting their success as a student.

Diversity, Equity and Inclusion - (213) 740-2101

Information on events, programs and training, the Provost's Diversity and Inclusion Council, Diversity Liaisons for each academic school, chronology, participation, and various resources for students.

USC Emergency - UPC: (213) 740-4321, HSC: (323) 442-1000 – 24/7 on call

Emergency assistance and avenue to report a crime. Latest updates regarding safety, including ways in which instruction will be continued if an officially declared emergency makes travel to campus infeasible.

<u>USC Department of Public Safety</u> - UPC: (213) 740-6000, HSC: (323) 442-1200 – 24/7 on call Non-emergency assistance or information.

Office of the Ombuds - (213) 821-9556 (UPC) / (323-442-0382 (HSC)

A safe and confidential place to share your USC-related issues with a University Ombuds who will work with you to explore options or paths to manage your concern.

Occupational Therapy Faculty Practice - (323) 442-2850 or otfp@med.usc.edu

Confidential Lifestyle Redesign services for USC students to support health promoting habits and routines that enhance quality of life and academic performance.