

**DSO 699: Fundamentals of Probability for
Data Science and Operations Research
(Fall 2024: Monday and Wednesday 12pm - 1:20pm)**

Syllabus (Version: March 4, 2024)

Contact Information

Instructor: Paat Rusmevichientong
E-mail: rusmevic@marshall.usc.edu
Office: BRI 400F
Office Hours: By appointment

Course Description: This is a course on the fundamentals of measure-theoretic probability for first and second-year PhD students in data sciences and operations. The treatment is mathematically rigorous and proof-based. The course is geared towards students who want to use probability in their research at a sophisticated level, including students reading and conducting research in stochastic control, reinforcement learning, statistics, operations research, queueing networks, revenue management, and the theory of machine learning.

Learning Objectives: Random variables occurs in many data science and operations research problems. The course will teach students how to rigorously analyze these random variables. At the end of this course, students will master the following learning objectives.

- Exhibit enough general mathematical “maturity” to write rigorous proofs of fundamental theorems in probability and stochastics.
- Identify random variables that arise in data science and operations applications and rigorously establish their distributions.
- Compute expectations of random variables and apply theorems from the class to establish properties of these expectations.
- Prove various forms of laws of large numbers and apply them.
- State and rigorously prove various forms of large deviation inequalities and identify applications where the inequalities can be used.
- Identify stochastic processes that arise in data science and operations management applications and rigorously establish properties of these processes.

Prerequisites: The course will assume familiarity with elementary undergraduate probability and some mathematical maturity. A course in real analysis is strongly encouraged but is not required.

Topics: The follows provide a rough outline of the topics that we will cover.

Measure Theoretic Probability: Sample spaces, σ -fields and probability measures, properties of probability measures (countably additive, union bound, inclusion-exclusion, continuity of probability), Caratheodory Extension Theorem, Lebesgue measure on $[0,1]$ and \mathbb{R} .

Random Variables: Random variables as measurable functions, σ -fields generated by random variables, discrete and continuous random variables, cumulative distribution functions, summary

statistics, moment generating functions, probability generating functions, examples of useful discrete and continuous random variables.

Independence: Independent random variables and Borel-Cantelli Lemma.

Integration and Expectation: Measure theoretic development, Fatou's Lemma, Dominated Convergence Theorem, product measure, Fubini's Theorem,

Different Modes of Convergence and Their Relationship: Almost sure convergence, convergence in probability, connections between a.s. and i.p. convergence, L_p convergence and uniform Integrability, convergence in distribution.

Laws of Large Numbers for Independent Random Variables: Strong Law of Large Number, Weak Law of Large Number, Chernoff bounds and other large deviation inequalities.

Applications: Probabilistic method, randomized algorithms, and high-dimensional probability

Introduction to Stochastic Processes: Bernoulli and Poisson processes, stationarity and memoryless property, stopping times.

Discrete-Time Markov Chains: Homogenous finite-state Markov chains, stationary distribution, recurrent and transience, Markov chains with a single recurrent class, uniqueness of stationary distribution, periodicity, mixing, absorption

Martingale Theory: Conditional expectation, martingales, martingale convergence and central limit theorems.

Grading: Participation 10%, Homework 40%, and Exams (Midterm and Final) 50%

Participation: Your participation grade is based in part on class attendance, being prepared for class, and constructive participation during class, including answering questions and contributing to discussions in class. The final participation score is at the discretion of the instructor, but it is my hope that everyone receives full participation credits.

Homework: The course will have 11 homework that are distributed throughout the semester. The due dates for the assignments are given in the course outline. Since there is no TA for the course, I will grade all of the homework submissions by myself. *All assignment solutions must be submitted online through Blackboard.* Moreover, all homework submissions must be **written up in LaTeX and submitted as PDF documents**. You are welcome to use your own LaTeX template. To ensure readability, please use at least a 12-point font and at least 1.5 line spacing. Sample LaTeX templates can be found at <https://www.overleaf.com/latex/templates>. No late homework will be accepted. The homework with the lowest score will be dropped.

Exams: We have two exams: midterm and final exams. We will give more weight to the exam with the higher score. The overall score for the exam will be computed based on the following formula:

(60% x maximum score between the two exams) + (40% x minimum score between the two exams)

References: About two-third of the materials will be based on the following textbook: *Probability Path* by Sidney I. Resnick. The remaining materials will be drawn from other references. There

are many excellent references with overlaps in terms of topics:

- *Probability and Random Processes* by Geoffrey Grimmett and David Stirzaker
- *Probability and Computing* by Michael Mitzenmacher and Eli Upfal
- *Probability Essentials* by Jean Jacod and Philip Protter
- *High-Dimensional Probability: An Introduction with Applications in Data Science* by Roman Vershynin

Course Disclaimer: The learning environment will be collaborative and supportive; we will learn from one another both in and out of the classroom. To that end, modifications to this syllabus may be warranted as determined by the instructor as we assess the learning needs of this particular class of students. In addition, grades for class participation are under the sole discretion of the instructor.

Tentative Course Outline

Week	Date	Lecture	Topic	Assignments
1	08/26	1	Measure Theoretic Probability I	
	08/28	2	Measure Theoretic Probability II	
2	09/02	3	No Class. Labor Day.	
	09/04	4	Random Variables I	HW1 is due
3	09/09	5	Random Variables II	
	09/11	6	Product Spaces and Independence I	HW2 is due
4	09/16	7	Product Spaces and Independence II	
	09/18	8	Integration and Expectation I	HW3 is due
5	09/23	9	Integration and Expectation II	
	09/25	10	Integration and Expectation III	HW4 is due
6	09/30	11	Integration and Expectation IV	
	10/02	12	Different Modes of Convergence I	HW5 is due
7	10/07	13	Different Modes of Convergence II	
	10/09	14	Different Modes of Convergence III	HW6 is due
8	10/14	15	Different Modes of Convergence IV	
	10/16	16	MIDTERM	
9	10/21	17	Laws of Large Numbers I	
	10/23	18	Laws of Large Numbers II	
10	10/28	19	Laws of Large Numbers III	
	10/30	20	Applications I	HW7 is due
11	11/04	21	Applications II	
	11/06	22	Introduction to Stochastic Processes I	HW8 is due
12	11/11	23	Introduction to Stochastic Processes II	
	11/13	24	Discrete-Time Markov Chains I	HW9 is due
13	11/18	25	Discrete-Time Markov Chains II	
	11/20	26	Martingale Theory I	HW10 is due
14	11/25	27	Martingale Theory II	
	11/27	28	No Class. Thanksgiving Holiday.	
15	12/02	29	Martingale Theory III	
	12/04	30	Martingale Theory IV	HW11 is due

Final Exam: Friday, December 13, 11am – 1pm

Academic Conduct, Learning Environment, and Support Systems

Academic Conduct: Plagiarism – presenting someone else’s ideas as your own, either verbatim or recast in your own words – is a serious academic offense with serious consequences. Please familiarize yourself with the discussion of plagiarism in SCampus in Part B, Section 11, “Behavior Violating University Standards” policy.usc.edu/scampus-part-b. Other forms of academic dishonesty are equally unacceptable. See additional information in SCampus and university policies on [Research and Scholarship Misconduct](#).

Policy on AI: The use of ChatGPT or any other AI software is strictly prohibited. Students are expected to complete all assignments using their own writing, analytical, and critical thinking skills. Any submission that is generated by AI will be considered a violation of academic honesty and will be treated as plagiarism. It is imperative that all work submitted in this course is entirely the student's own, without any involvement of AI-generated content. Any unauthorized use of AI tools will be subject to disciplinary action in accordance with USC’s Academic Integrity and Student Conduct Policies.

Open Expression and Respect for All: An important goal of the educational experience at USC Marshall is to be exposed to and discuss diverse, thought-provoking, and sometimes controversial ideas that challenge one’s beliefs. In this course we will support the values articulated in the USC Marshall [“Open Expression Statement.”](#)

Students with Accommodations: USC welcomes students with disabilities into all of the University’s educational programs. The Office of Student Accessibility Services (OSAS) is responsible for the determination of appropriate accommodations for students who encounter disability-related barriers. Once a student has completed the OSAS process (registration, initial appointment, and submitted documentation) and accommodations are determined to be reasonable and appropriate, a Letter of Accommodation (LOA) will be available to generate for each course. The LOA must be given to each course instructor by the student and followed up with a discussion. This should be done as early in the semester as possible as accommodations are not retroactive. More information can be found at osas.usc.edu. You may contact OSAS at (213) 740-0776 or via email at osasfrontdesk@usc.edu.

Support Systems:

Counseling and Mental Health - (213) 740-9355 – 24/7 on call
sites.google.com/usc.edu/counseling-mental-health

Free and confidential mental health treatment for students, including short-term psychotherapy, group counseling, stress fitness workshops, and crisis intervention.

988 Suicide and Crisis Lifeline - 988 for both calls and text messages – 24/7 on call
988lifeline.org

The 988 Suicide and Crisis Lifeline (formerly known as the National Suicide Prevention Lifeline) provides free and confidential emotional support to people in suicidal crisis or emotional distress 24 hours a day, 7 days a week, across the United States. The Lifeline is comprised of a national network of over 200 local crisis centers, combining custom local care and resources with national standards and best practices. The new, shorter phone number makes it easier for people to remember and access mental health crisis services (though the previous 1 (800) 273-8255 number will continue to function indefinitely) and represents a continued commitment to those in crisis.

Relationship and Sexual Violence Prevention Services (RSVP) - (213) 740-9355(WELL) – 24/7 on call
sites.google.com/usc.edu/rsvpclientservices/home

Free and confidential therapy services, workshops, and training for situations related to gender- and power-based harm (including sexual assault, intimate partner violence, and stalking).

Office for Equity, Equal Opportunity, and Title IX (EEO-TIX) - (213) 740-5086
eeotix.usc.edu

Information about how to get help or help someone affected by harassment or discrimination, rights of protected classes, reporting options, and additional resources for students, faculty, staff, visitors, and applicants.

Reporting Incidents of Bias or Harassment - (213) 740-5086 or (213) 821-8298

usc-advocate.symplicity.com/care_report

Avenue to report incidents of bias, hate crimes, and microaggressions to the Office for Equity, Equal Opportunity, and Title for appropriate investigation, supportive measures, and response.

The Office of Student Accessibility Services (OSAS) - (213) 740-0776

osas.usc.edu

OSAS ensures equal access for students with disabilities through providing academic accommodations and auxiliary aids in accordance with federal laws and university policy.

USC Campus Support and Intervention - (213) 740-0411

campussupport.usc.edu

Assists students and families in resolving complex personal, financial, and academic issues adversely affecting their success as a student.

Diversity, Equity and Inclusion - (213) 740-2101

diversity.usc.edu

Information on events, programs and training, the Provost's Diversity and Inclusion Council, Diversity Liaisons for each academic school, chronology, participation, and various resources for students.

USC Emergency - UPC: (213) 740-4321, HSC: (323) 442-1000 – 24/7 on call

emergency.usc.edu

Emergency assistance and avenue to report a crime. Latest updates regarding safety, including ways in which instruction will be continued if an officially declared emergency makes travel to campus infeasible.

USC Department of Public Safety - UPC: (213) 740-6000, HSC: (323) 442-1200 – 24/7 on call

dps.usc.edu

Non-emergency assistance or information.

Office of the Ombuds - (213) 821-9556 (UPC) / (323-442-0382 (HSC)

ombuds.usc.edu

A safe and confidential place to share your USC-related issues with a University Ombuds who will work with you to explore options or paths to manage your concern.

Occupational Therapy Faculty Practice - (323) 442-2850 or otfp@med.usc.edu

chan.usc.edu/patient-care/faculty-practice

Confidential Lifestyle Redesign services for USC students to support health promoting habits and routines that enhance quality of life and academic performance.