Math 606 in Summer 2024: Extreme Values and Rare Events

Class number 054–39482R
Class meetings: MW, 9:30am-12:30pm, KAP 163.

Information on this and related pages changes frequently.

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Office Hours: MW after the class. Appointments at other time are welcome.

Course objective: to get used to mathematical tools for quantifying and modeling extreme and/or unlikely behavior.

More general goal: to learn something interesting, new, and/or useful.

Official grading scheme: 20% class participation, 40% homework assignments, 40% final presentation.

Main reference: Sidney I. Resnick. Extreme values, regular variation and point processes. Reprint of the 1987 original. Springer Series in Operations Research and Financial Engineering, *Springer, New York,* 2008. xiv+320 pp. ISBN: 978-0-387-75952-4

This book is indeed **three-in-one**: Regular Variation Point Processes

A longer list of references

The People Of Extreme Values

Homework problems and more

An example of a book review from *Math reviews* [Edition 1, Edition 2] **and from** the Bulletin of the AMS

My notes

- Basic inequalities
- Gaussian objects: Normal random variables, CLT, and more
- A summary of Brownian motion

- Lambert's W function
- Gamma and Beta Functions
- Cauchy's functional equation
- Cauchy distribution
- About harmonic numbers
- A summary of renewal theory
- Markov processes: A summary of continuous-time Markov processes, A summary of continuous time Markov chains
- Poisson process: definition, arrival times, waiting times
- Stochastic analysis in continuous time
- Weak convergence of probability measures
- LDP and rare events
- Extremes
- A summary of large deviations
- A summary of the Cameron-Martin-Girsanov theorem and related results

Other notes

- (Almost) everything you need to know about probability distributions
- Coupon collection problem (an example where an extreme value distribution can appear in the limit)
- Determinant point processes
- All about Cauchy's functional equation
- Lambert's W function
- The classical paper about associated random variables
- The Pickands-Balkema-de Haan theorem: the original papers by Pichands and Balkema and De Haan, and a more modern treatment

Our progress.

May 15. An overview of the class, the book, and some foundational material from real analysis and probability; three types of extreme value distributions.
Related material: Convergence of random variables and an illustration

May 20. Convergence of max of iid-s: normalization, domains of attraction, generalized extreme value distribution.
May 22. Beyond convergence in distribution: convergence of moments, convergence of pdf-s, convergence of the tails, rate of convergence in the Kolmogorov metric.

May 27. No class (Memorial Day, University Holiday)
May 29. Random measures, weak and vague convergence, Poisson random measure.

June 3. Record times, record values, and related concepts: definitions, examples, and asymptotics; a brief overview of the Skorokhod space D.
June 5. Some extensions to non-iid case and to vector case; a few words about copula and association.

June 10. Statistical aspects of extreme value theory: Pickands-Balkema-de Haan theorem, tail index estimation, prediction of records.
June 12. Large Deviations Principle, theorems of Cramer and Schilder, and applications.

June 17. Importance sampling: motivation, general idea, a concrete connection with large deviations, and some examples.
June 19. No class (Juneteenth, Non-Instructional Day)

June 24. More on random point measures: construction of the n-point correlation function and a few examples.
June 26. Determinantal point processes and the distribution of the largest eigenvalue in a GUE.

July 1. Concluding discussion: what did you learn, what would you like to lean in more detail, what was your favorite homework problem?