EE 592: Computational Methods for Inverse Problems
Spring 2024

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University of Southern California

Lectures: 9:00-10:50am Tue/Thu, KAP 144

First Class: Tuesday, January 9th
Midterm: Thursday, February 29th
Last Class: Thursday, April 25th
Final Exam: Tuesday, May 7th, 8am-10am

Instructor: Professor Justin P. Haldar
Email: jhaldar@usc.edu
Office: EEB 442 & Zoom (see website and Lecture 01 slides)
Office Hours: 2:00-3:30pm Tue/Wed

Catalog Description: A rigorous description of vector space and functional analysis concepts and tools that are useful for solving inverse problems in real-world applications.

Prerequisites: EE 483 and EE 510

Course Overview: In many practical applications, we are interested in estimating a signal of interest based on measured data. For example, we may be interested in problems such as: estimating a 3D medical image from a set of 2D X-ray projections; estimating a high-quality picture from data acquired with a noisy/blurry/low-resolution camera; reconstructing a bandlimited signal from non-uniform samples and/or using a sampling rate that violates the Nyquist criterion; estimating the speech produced by one specific person from an audio recording of a noisy party where many people are talking simultaneously; or estimating the location of an earthquake based on information measured by an array of seismometers. These are all examples of "inverse problems," and these types of problems are found everywhere in the modern world.

This course provides a rigorous description of vector space and functional analysis concepts and tools that are commonly used to solve modern inverse problems in a variety of real-world applications. Topics will include linear inverse problems in finite and infinite dimensional vector spaces; the existence, uniqueness, and stability of solutions to inverse problems; ill-posedness, conditioning, and regularization; Banach and Hilbert spaces; optimal design of experiments; iterative optimization methods for solving large-scale and/or nonlinear inverse problems; sparse and low-rank signal modeling; and harmonic retrieval. While the course material focuses on inverse problems, the concepts, tools, and methods we discuss are also useful for solving signal approximation, signal representation, and signal design problems, and are broadly relevant to signal processing, machine learning, and optimization.

Coursework will include proving theorems, deriving methods and algorithms for solving inverse problems, and the practical application of these methods to real-world problems. Real-world application examples will be far-ranging, including artifact and noise removal in audio, image, and video signals; computational imaging (i.e., forming an image from low-quality measurements or indirect measurements); the design of optimal sensing systems; and direction-of-arrival estimation from sensor array data. Students will get extensive experience applying these techniques to a variety of practical problems in homework assignments, and will also undertake a project on a topic of their choice.

Required Texts: None.

Recommended Texts:

- D. Luenberger, *Optimization by Vector Space Methods*, Wiley, 1969.
- T. Moon and W. Sterling, *Mathematical Methods and Algorithms for Signal Processing*, Prentice Hall, 2000.
- J. A. Fessler. *Image reconstruction: Algorithms and analysis.* Unpublished book manuscript.
- C. A. Bouman. *A Guide to the Tools of Model Based Image Processing.* Unpublished book manuscript.

Grading and Course Policies:

40% Homeworks
20% Project
20% Midterm Exam
20% Final Exam

Homework must be submitted electronically by 9:00pm PST on the due date. Late submissions will receive a score of zero. The final homework grade will be based on your average score after discarding the lowest.

Students are allowed (and encouraged!) to discuss homework assignments with fellow classmates, but are expected to complete homework assignments individually. USC's recommended sanction for plagiarism, unauthorized collaboration, and/or cheating on any coursework is an F for the course, with a possibility for further disciplinary action. Accessing EE592 course materials from previous semesters or giving others access to EE592 course materials will be viewed as academic misconduct.

Several of the homeworks will require MATLAB or Python programming. It is your responsibility to make sure that you know how to access the software and read/write/debug code.

All exams are cumulative and closed book, with no calculators (subject to change). Please check now for any conflicts with the scheduled exam times.

Websites:
All course materials will be distributed through the USC Blackboard website: https://blackboard.usc.edu/. It is your responsibility to check the website regularly for updates (notes, assignments, due dates, etc.).

We will be using Piazza for class discussion. The system is aimed at getting students help from classmates and instructors quickly and efficiently. Rather than emailing questions to the instructor, students are encouraged to post your questions on Piazza so that everyone in the course can see the conversation and benefit from the discussion. The Piazza page for the course can be found at: https://piazza.com/usc/spring2024/ee592/home. If you have any problems or feedback for the developers, email team@piazza.com.

Suggestions:

My goal is to teach you and your classmates as much as possible about solving inverse problems, while simultaneously inspiring your interest, excitement, and curiosity about the material. This will be easier if you:

- Come to class on time and pay attention.
- Ask questions and participate in classroom discussion.
- Do all of the assignments.
- Make use of office hours.
- If you're struggling with the material, don't wait until the last minute to talk to us.

COURSE OUTLINE

Week 1: Inverse problems; analytic versus model-based solution approaches; least squares, maximum likelihood, penalized maximum likelihood, maximum *a posteriori*, minimum mean-squared error, and minimum absolute error estimation; linear vector spaces and subspaces; linear varieties, linear combinations, and linear independence; bases.
Week 2: Finite and infinite dimensional spaces; norms; existence and uniqueness of solutions in $\mathbb{C}^N$; left and right inverses; orthogonality.
Week 3: Orthogonality principle; projectors; least-squares solutions.
Week 4: Minimum norm solutions; minimum norm least-squares solutions; Moore-Penrose pseudoinverse; singular value decomposition,
Week 5: Matrix norms and inequalities; Eckart-Young theorem and applications.

Week 6: Sensitivity and conditioning of $\mathbf{Ax} = \mathbf{b}$ with errors in both $\mathbf{A}$ and $\mathbf{b}$; sensitivity and conditioning of minimum norm least-squares and least-squares problems; regularization; SVD filtering; Tikhonov regularization.

Week 7: Total least squares and applications; Landweber iteration; conjugate gradient method.

Week 8: Nonlinear regularization and general optimization principles; MIDTERM.

Week 9: Majorize-minimize algorithms.

Week 10: Sparsity-constrained inverse problems in $\mathbb{C}^N$; proofs of perfect reconstruction for ℓ_0 and ℓ_1 minimization under restricted isometry conditions; low-rank matrix completion; constrained optimization; penalty method; augmented Lagrangian method; ADMM.

Week 11: Hamel bases, normed vector spaces, the $\ell_p(\mathbb{Z}^N)$ and $\mathcal{L}_p(\mathbb{R}^N)$ vector spaces; equivalence classes; inner product spaces; induced norms; parallelogram law.

Week 12: Polarization identity; linear operators; norms on linear operators; adjoints; matrix representations of inverse problems in finite dimensional spaces; convergence of vector sequences; vector Cauchy sequences; Banach spaces; Hilbert spaces.

Week 13: Minimum norm least-squares problems in Hilbert spaces; Gauss-Markov theorem; optimal experiment design.

Week 14: Harmonic retrieval and applications; variable projection.

Week 15: Markov random fields; Gibbs distributions; Hammersley-Clifford theorem; project presentations.

Week 16: FINAL EXAMINATION

Academic Integrity

The University of Southern California is foremost a learning community committed to fostering successful scholars and researchers dedicated to the pursuit of knowledge and the transmission of ideas. Academic misconduct is in contrast to the university's mission to educate students through a broad array of first-rank academic, professional, and extracurricular programs and includes any act of dishonesty in the submission of academic work (either in draft or final form).

This course will follow the expectations for academic integrity as stated in the USC Student Handbook. All students are expected to submit assignments that are original work and prepared specifically for the course/section in this academic term. You may not submit work written by others or "recycle" work prepared for other courses without obtaining written permission from the instructor(s). Students suspected of engaging in academic misconduct will be reported to the Office of Academic Integrity.

Other violations of academic misconduct include, but are not limited to, cheating, plagiarism, fabrication (e.g., falsifying data), knowingly assisting others in acts of academic dishonesty, and any act that gains or is intended to gain an unfair academic advantage.

The impact of academic dishonesty is far-reaching and is considered a serious offense against the university and could result in outcomes such as failure on the assignment, failure in the course, suspension, or even expulsion from the university.

For more information about academic integrity see the student handbook or the Office of Academic Integrity's website, and university policies on Research and Scholarship Misconduct.

Students and Disability Accommodations:

USC welcomes students with disabilities into all of the University's educational programs. The Office of Student Accessibility Services (OSAS) is responsible for the determination of appropriate accommodations for students who encounter disability-related barriers. Once a student has completed the OSAS process (registration, initial appointment, and submitted documentation) and accommodations are determined to be reasonable and appropriate, a Letter of Accommodation (LOA) will be available to generate for each course. The LOA must be given to each course instructor by the student and followed up with a discussion. This should be done as early in the semester as possible as accommodations are not retroactive. More information can be found at osas.usc.edu. You may contact OSAS at (213) 740-0776 or via email at osasfrontdesk@usc.edu.

Support Systems:

Counseling and Mental Health - (213) 740-9355 – 24/7 on call
Free and confidential mental health treatment for students, including short-term psychotherapy, group counseling, stress fitness workshops, and crisis intervention.

988 Suicide and Crisis Lifeline - 988 for both calls and text messages – 24/7 on call
The 988 Suicide and Crisis Lifeline (formerly known as the National Suicide Prevention Lifeline) provides free and confidential emotional support to people in suicidal crisis or emotional distress 24 hours a day, 7 days a week, across the United States. The Lifeline is comprised of a national network of over 200 local crisis centers, combining custom local care and resources with national standards and best practices. The new, shorter phone number makes it easier for people to remember and access mental health crisis services (though the previous 1 (800) 273-8255 number will continue to function indefinitely) and represents a continued commitment to those in crisis.

Relationship and Sexual Violence Prevention Services (RSVP) - (213) 740-9355(WELL) – 24/7 on call
Free and confidential therapy services, workshops, and training for situations related to gender- and power-based harm (including sexual assault, intimate partner violence, and stalking).

Office for Equity, Equal Opportunity, and Title IX (EEO-TIX) - (213) 740-5086
Information about how to get help or help someone affected by harassment or discrimination, rights of protected classes, reporting options, and additional resources for students, faculty, staff, visitors, and applicants.

Reporting Incidents of Bias or Harassment - (213) 740-5086 or (213) 821-8298

Avenue to report incidents of bias, hate crimes, and microaggressions to the Office for Equity, Equal Opportunity, and Title for appropriate investigation, supportive measures, and response.

The Office of Student Accessibility Services (OSAS) - (213) 740-0776
OSAS ensures equal access for students with disabilities through providing academic accommodations and auxiliary aids in accordance with federal laws and university policy.

USC Campus Support and Intervention - (213) 740-0411
Assists students and families in resolving complex personal, financial, and academic issues adversely affecting their success as a student.

Diversity, Equity and Inclusion - (213) 740-2101
Information on events, programs and training, the Provost's Diversity and Inclusion Council, Diversity Liaisons for each academic school, chronology, participation, and various resources for students.

USC Emergency - UPC: (213) 740-4321, HSC: (323) 442-1000 – 24/7 on call
Emergency assistance and avenue to report a crime. Latest updates regarding safety, including ways in which instruction will be continued if an officially declared emergency makes travel to campus infeasible.

USC Department of Public Safety - UPC: (213) 740-6000, HSC: (323) 442-1200 – 24/7 on call
Non-emergency assistance or information.

Office of the Ombuds - (213) 821-9556 (UPC) / (323-442-0382 (HSC)
A safe and confidential place to share your USC-related issues with a University Ombuds who will work with you to explore options or paths to manage your concern.

Occupational Therapy Faculty Practice - (323) 442-2850 or otfp@med.usc.edu
Confidential Lifestyle Redesign services for USC students to support health promoting habits and routines that enhance quality of life and academic performance.