GEOL 558: Inverse Theory in the Earth Sciences

Description

Units: 3
Term: Spring 2024
Lectures: Wed 2:00-4:20 pm
Location: ZHS 118 (Zumberge Hall, first floor)
Instructor: Sylvain Barbot (sbarbot@usc.edu)
Office Hours: (upon appointment).

Course Description

The course provides an introduction to inverse methods applicable to a wide range of scientific problems. The class will introduce the Bayesian description of inverse problems and various optimization methods based on a Hilbert space. In addition, the class will cover practical approaches to solve typical inverse problems. We will introduce important concepts, such as non-uniqueness, resolution, and model uncertainties. The class also describe approaches to time-dependent inverse problems, such as Kalman filters and particle filters.

Learning Objectives

By the end of this course, the students will be introduced to a quantitative description of forward and inverse problems and practical approaches to solving them.

Prerequisites:

Recommended preparation: GEOL-425L “Data Analysis in the Earth and Environmental Sciences”. Working knowledge of algebra, calculus, and statistics is useful.

Communication

Regular communication will be conducted through Blackboard (https://blackboard.usc.edu).

Lectures

Spring recess is March 10-17.

Examinations

The grades will be based on attendance (25%), assignments (50%), and a final project (25%).

Required Materials

Access to a Posix computer with scientific computing tools (Matlab, Python, Julia) is required.

Assignments

Some work will be assigned weekly.

Classroom norms

Student participation during lecture is strongly encouraged. Always feel free to ask questions and clarifications.

Lecture content

Forward problems

- Discretization of integral equations
- Setup of forward and inverse problems

Elements of statistics

- Single-variate probability
- Probability density function
- Normal distribution (Log-normal distribution, Chi-square distribution with one degree of freedom, Chi-square distribution with k degrees of freedom, Exponential distribution, Pareto distribution)
- Cumulative density function
- Change of variable
- Characteristic function
- Sums of independent random variables
- Central limit theorem
- Maximum-likelihood estimation
- Monte Carlo sampling

Multi-variate probability

- Joint probabilities
- Independent variables
- Moments and covariance matrix
- Correlation coefficient
- N-dimensional normal distribution
- Conditional probability
- Marginal probability
- Change of variables
- Sums of independent random variables
- Products and ratios of independent random variables
- The Cauchy distribution
- Monte Carlo sampling of multi-variate probability distributions

Inverse Theory

- Definitions
- Homogeneous probability distributions
- Jeffrey's parameters
- Homogeneous distribution on a sphere
- Conjunction of probabilities

Definition of the inverse problem

- Joint prior information

- Theoretical probability density function
- Conjunction of information
- Solutions of the inverse problem
- Normal distribution of observations and predictions
- Normal prior information

Linear forward model

- The design matrix
- Resolution operator
- Uniform prior information
- Regularization by smoothing

Optimization

- Norms
- Over-determined least-squares
- Under-determined least-squares
- Other minimizations with constraints
- Pseudo inverse
- Model resolution matrix
- Data resolution matrix
- Stability of inverse solutions
- Tikhonov regularization

Kalman filter

- Filtering
- Regularized Kalman filter
- Rauch-Tung-Striebel smoothing

Particle filter

- Monte-Carlo sampling of large multi-variate probability density functions

Lecture content is subject to change without warning.

Textbooks

Aster, R.C., Borchers, B. and Thurber, C.H., 2018. *Parameter estimation and inverse problems*. Elsevier.

Parker, R.L., 1994. *Geophysical inverse theory* (Vol. 1). Princeton university press.

Tarantola, A., 2005. *Inverse problem theory and methods for model parameter estimation*. Society for Industrial and Applied Mathematics.