



**CSCI 699: Mathematical Foundations of  
Intelligent Autonomy**

**Units: 4.0**

**Fall 2023 TuTh 4-5:50pm**

**Location:** SOS B37

**Instructor:** Lars Lindemann

**Office:** SAL 328

**Office Hours:** Every Monday at 11 am

**Contact Info:** llindema@usc.edu (replies can be expected  
within 48 hours during weekdays)

## Course Description

Autonomous systems are engineered systems that operate without human intervention. Applications can be found in self-driving cars, intelligent transportation, and robotics, e.g., to control mobile robot fleets to deliver food in Santa Monica. Autonomous systems are usually safety-critical, e.g., avoiding collisions with pedestrians, while tasked to accomplish complex system specifications, e.g., delivering food in pre-defined orders within fixed time intervals. This raises fundamental question regarding the safety and reliability of these increasingly complex and intelligent systems that we aim to understand in this course.

Autonomous systems research has its foundations in dynamical systems theory. Systems theory deals with complex systems but is limited in scope by only dealing with simple stability and invariance specifications. Formal methods are mathematical techniques for the specification, design, and verification of software and hardware systems. These techniques are formal as system specifications are grounded in mathematical logic that can express intelligent system behavior and that allow for a formal deduction of system correctness in that logic. Specifically, temporal logics are used as they extend Boolean logic with temporal modalities to express truly complex spatial and temporal system requirements. Over the past decade, a new community has formed that works at the intersection of formal methods and systems & control theory to design intelligent autonomous systems that satisfy temporal logic specifications.

In this course, we will study basic concepts from the fields of dynamical systems theory (state space representation, stability, forward invariance, control theory) and formal methods (system abstractions, temporal logics, temporal/spatial robustness, stochastic temporal logics). We then use ideas from these fields to develop formal control design techniques using mixed integer linear programming and time-varying control barrier functions. The final part of the course will contain a project in which students can implement the learned tools on an autonomous systems simulator of their choice.

Intended audience: any graduate student broadly interested in autonomous systems research, and more specifically with interests in control & dynamical systems, formal methods, and optimization.

## Learning Objectives and Outcomes

After taking the course, students will know basic concepts related to verifying and designing autonomous systems. Particularly, the students will learn basic concepts from systems theory (state space representation, stability, forward invariance, control theory) and formal methods (system abstractions, temporal logics, temporal/spatial robustness, stochastic temporal logics). The students will further learn how to bridge ideas from these two fields and be able to design formal control algorithms (mixed integer linear programming and time-varying control barrier functions) and apply them to autonomous systems of their choice. Throughout the course, students will learn to critically assess limitations of the learned techniques.

## Recommended Preparation

Due to the interdisciplinary nature of the topic, the course will provide introductions to the fields of dynamical systems theory and formal methods. As such, no strict prerequisites are required. After all, the most important prerequisite is the student's interest in the topic, motivation, and commitment to learning. Nonetheless, some background in systems theory, optimization, and/or logic may be helpful, e.g., as (partially) taught in

- convex and combinatorial optimization (CS 675) or optimization for information and data sciences (EE 588),
- linear control systems (EE482), linear systems theory (EE 585), or nonlinear and adaptive control (EE 587),
- introduction to artificial intelligence (CSCI 360).

## Course Notes

The final grade will be determined based on attendance, three homeworks, a course project, and a final exam. Lecture notes and slides will be posted online after each lecture. The lectures will not be recorded or broadcasted online.

## Technological Proficiency and Hardware/Software Required

Course homeworks and projects require standard computing software (Matlab, Python, C++).

## Required Readings and Supplementary Materials

Required readings and supplementary materials will be announced during the course.

## Description and Assessment of Assignments

The three homeworks will contain problem sets based on the given lectures. The exam will similarly be based on the lectures. The course project is a research project, i.e., it is the students responsibility to i) define a problem with a scope aligned with the course topic, and ii) to propose a viable solution and illustrate it. It is required that students implement their solution on an autonomous systems simulator of their choice (the course instructor will provide suggestions on suitable simulators). A final project report has to be submitted and a 10-15 minute research presentation has to be given in front of the class. The course project can be performed in teams of at most two students.

## Grading Breakdown

Including the above detailed assignments, how will students be graded overall? Participation should be no more than 15%, unless justified for a higher amount. All must total 100%.

Assignment	Points	% of Grade
Participation	10	10
Homework 1	10	10
Homework 2	10	10
Homework 3	10	10
Exam	30	30
Course Project	30	30
<b>TOTAL</b>	<b>100</b>	<b>100</b>

## Assignment Submission Policy

TBA

## Grading Timeline

TBA

## Additional Policies

TBA: Add any additional policies that students should be aware of: late assignments, missed classes, attendance expectations, use of technology in the classroom, etc.

### Course Schedule: A Weekly Breakdown

	Topics/Daily Activities	Readings and Homework	Deliverable/ Due Dates
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<b>Week 1</b>	Course Overview, Linear Systems (state space representation, stability, control)	Chapters 2, 4.1, 4.2, 5.1, 5.4, 6.1, 6.2, 8.1, and 8.2 in [1]	
<b>Week 2</b>	Nonlinear Systems (stability and Lyapunov functions)	Chapters 1, 4.1 and 4.3 in [2] Chapter 2.1 in [3]	
<b>Week 3</b>	Nonlinear Systems (forward invariance and barrier functions)	Chapter 2.1 in [3] Research paper [4]	
<b>Week 4</b>	Nonlinear Systems (Lyapunov functions via learning and sum-of-squares programming, feedback control)	Research paper [4] Chapter 2.2 in [3] <b>Homework 1 out (on Friday)</b>	
<b>Week 5</b>	Nonlinear Systems (control barrier and Lyapunov functions, guest lecture on systems learning) + Course Project	Chapter 2.2 in [3]	
<b>Week 6</b>	Formal Methods (system abstraction, linear temporal logic, model checking)	Chapter 3.1 in [3] Parts I and II in [5]	<b>Homework 1 (on Friday)</b>
<b>Week 7</b>	Formal Methods (signal temporal logic, monitoring, spatial robustness)	Research paper [6] and [7] Chapter 3.2 in [3]	
<b>Week 8</b>	Formal Methods (temporal robustness, stochastic temporal logics)	Research paper [6], [8], and [9] <b>Homework 2 out (on Friday)</b>	
<b>Week 9</b>	Formal Control Synthesis (mixed integer linear programming)	Research papers [10] and [11]	<b>Homework 2 (on Friday)</b>
<b>Week 10</b>	Formal Control Synthesis (time-varying control barrier functions)	Research paper [12] Chapter 5 in [3]	
<b>Week 11</b>	Formal Control Synthesis (scalable synthesis for multi-agent systems)	Research papers [13] and [14] Chapter 6 in [3] <b>Homework 3 out (on Friday)</b>	
<b>Week 12</b>	Formal Methods (guest lecture on logic learning) + Course Project		<b>Homework 3 (on Friday)</b>
<b>Week 13</b>	Project Days		
<b>Week 14</b>	Project Days		
<b>Week 15</b>	Project Presentations		Project Presentation
<b>FINAL</b>	Final Project Report		<i>Due on the university-scheduled date of the final exam</i>

## References

- [1] "Linear System Theory and Design" by Chi-Tsong Chen
- [2] "Nonlinear Systems" by Hassan Khalil
- [3] "Formal Methods for Multi-Agent Control Systems" by Lars Lindemann and Dimos V. Dimarogonas
- [4] "A framework for worst-case and stochastic safety verification using barrier functions" by Stephen Prajna, Ali Jadbabaie, and George Pappas
- [5] "Formal Methods for Discrete-Time Systems" by Calin Belta, Boyan Yordanov, and Ebru Gol
- [6] "Robust Satisfaction of Temporal Logic over Real-Valued Signals" by Alexandre Donze and Oded Maler
- [7] "Robustness of Temporal Logic Specifications for Continuous-Time Signals" by Georgios Fainekos and George Pappas
- [8] "Temporal Robustness of Stochastic Signals" by Lars Lindemann, Alena Rodionova, and George Pappas
- [9] "Risk of Stochastic Systems for Temporal Logic Specifications" by Lars Lindemann, Lejun Jiang, Nikolai Matni, and George Pappas
- [10] "Model Predictive Control for Signal Temporal Logic Specification" by Vasumathi Raman et al.
- [11] "Time-robust control for STL Specifications" by Alena Rodionova, Lars Lindemann, Manfred Morari, and George Pappas
- [12] "Control Barrier Functions for Signal Temporal Logic Tasks" by Lars Lindemann and Dimos Dimarogonas
- [13] "Control Barrier Functions for Multi-Agent Systems under Conflicting Local Signal Temporal Logic Tasks" by Lars Lindemann and Dimos Dimarogonas
- [14] "Barrier Function Based Collaborative Control of Multiple Robots under Signal Temporal Logic Tasks" by Lars Lindemann and Dimos Dimarogonas

## Statement on Academic Conduct and Support Systems

### **Academic Integrity:**

The University of Southern California is a learning community committed to developing successful scholars and researchers dedicated to the pursuit of knowledge and the dissemination of ideas. Academic misconduct, which includes any act of dishonesty in the production or submission of academic work, comprises the integrity of the person who commits the act and can impugn the perceived integrity of the entire university community. It stands in opposition to the university's mission to research, educate, and contribute productively to our community and the world.

All students are expected to submit assignments that represent their own original work, and that have been prepared specifically for the course or section for which they have been submitted. You may not submit work written by others or "recycle" work prepared for other courses without obtaining written permission from the instructor(s).

Other violations of academic integrity include, but are not limited to, cheating, plagiarism, fabrication (e.g., falsifying data), collusion, knowingly assisting others in acts of academic dishonesty, and any act that gains or is intended to gain an unfair academic advantage.

The impact of academic dishonesty is far-reaching and is considered a serious offense against the university. All incidences of academic misconduct will be reported to the Office of Academic Integrity and could result in outcomes such as failure on the assignment, failure in the course, suspension, or even expulsion from the university.

For more information about academic integrity see [the student handbook](#) or the [Office of Academic Integrity's website](#), and university policies on [Research and Scholarship Misconduct](#).

Please ask your instructor if you are unsure what constitutes unauthorized assistance on an exam or assignment, or what information requires citation and/or attribution.

### **Students and Disability Accommodations:**

USC welcomes students with disabilities into all of the University's educational programs. The Office of Student Accessibility Services (OSAS) is responsible for the determination of appropriate accommodations for students who encounter disability-related barriers. Once a student has completed the OSAS process (registration, initial appointment, and submitted documentation) and accommodations are determined to be reasonable and appropriate, a Letter of Accommodation (LOA) will be available to generate for each course. The LOA must be given to each course instructor by the student and followed up with a discussion. This should be done as early in the semester as possible as accommodations are not retroactive. More information can be found at [osas.usc.edu](http://osas.usc.edu). You may contact OSAS at (213) 740-0776 or via email at [osasfrontdesk@usc.edu](mailto:osasfrontdesk@usc.edu).

### **Support Systems:**

[Counseling and Mental Health](#) - (213) 740-9355 – 24/7 on call

Free and confidential mental health treatment for students, including short-term psychotherapy, group counseling, stress fitness workshops, and crisis intervention.

[988 Suicide and Crisis Lifeline](#) - 988 for both calls and text messages – 24/7 on call

The 988 Suicide and Crisis Lifeline (formerly known as the National Suicide Prevention Lifeline) provides free and confidential emotional support to people in suicidal crisis or emotional distress 24 hours a day, 7 days a week, across the United States. The Lifeline is comprised of a national network of over 200 local crisis centers, combining custom local care and resources with national standards and best practices. The new, shorter phone number makes it easier for people to remember and access mental health crisis services (though the previous 1 (800) 273-8255 number will continue to function indefinitely) and represents a continued commitment to those in crisis.

[Relationship and Sexual Violence Prevention Services \(RSVP\)](#) - (213) 740-9355(WELL) – 24/7 on call

Free and confidential therapy services, workshops, and training for situations related to gender- and power-based harm (including sexual assault, intimate partner violence, and stalking).

[Office for Equity, Equal Opportunity, and Title IX \(EEO-TIX\)](#) - (213) 740-5086

Information about how to get help or help someone affected by harassment or discrimination, rights of protected classes, reporting options, and additional resources for students, faculty, staff, visitors, and applicants.

[Reporting Incidents of Bias or Harassment](#) - (213) 740-5086 or (213) 821-8298

Avenue to report incidents of bias, hate crimes, and microaggressions to the Office for Equity, Equal Opportunity, and Title for appropriate investigation, supportive measures, and response.

[The Office of Student Accessibility Services \(OSAS\)](#) - (213) 740-0776

OSAS ensures equal access for students with disabilities through providing academic accommodations and auxiliary aids in accordance with federal laws and university policy.

[USC Campus Support and Intervention](#) - (213) 740-0411

Assists students and families in resolving complex personal, financial, and academic issues adversely affecting their success as a student.

[Diversity, Equity and Inclusion](#) - (213) 740-2101

Information on events, programs and training, the Provost's Diversity and Inclusion Council, Diversity Liaisons for each academic school, chronology, participation, and various resources for students.

[USC Emergency](#) - UPC: (213) 740-4321, HSC: (323) 442-1000 – 24/7 on call

Emergency assistance and avenue to report a crime. Latest updates regarding safety, including ways in which instruction will be continued if an officially declared emergency makes travel to campus infeasible.

[USC Department of Public Safety](#) - UPC: (213) 740-6000, HSC: (323) 442-1200 – 24/7 on call

Non-emergency assistance or information.

[Office of the Ombuds](#) - (213) 821-9556 (UPC) / (323-442-0382 (HSC)

A safe and confidential place to share your USC-related issues with a University Ombuds who will work with you to explore options or paths to manage your concern.

[Occupational Therapy Faculty Practice](#) - (323) 442-2850 or [otfp@med.usc.edu](mailto:otfp@med.usc.edu)

Confidential Lifestyle Redesign services for USC students to support health promoting habits and routines that enhance quality of life and academic performance.