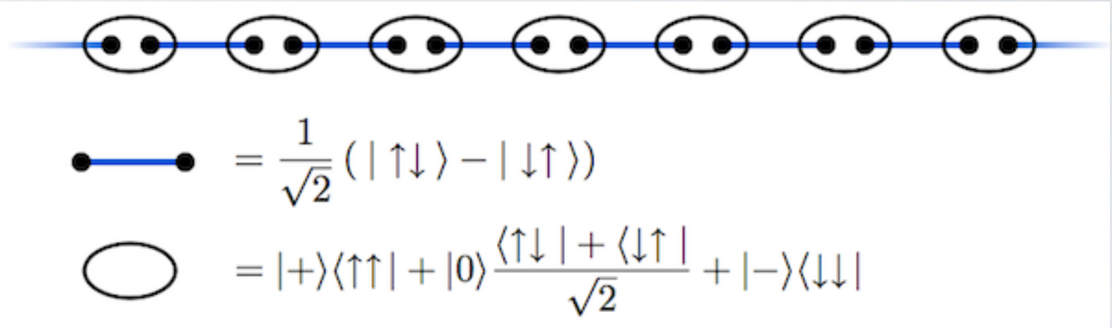
INTRODUCTION TO TOPOLOGICAL PHASES (PHYS 720)

Spring 2023

The Landau paradigm, in which different phases of matter can be related to different subgroups of a given group (i.e. symmetry breaking) and are therefore characterized by different local order parameters, has long been a stalwart guiding and organizing principle in physics.
However, the (not so recent) discovery of **topological phases** has opened a new chapter in this story. In such phases, the many-body wave functions cannot be described with purely localized orbitals, and the information is stored non-locally, spread over the entire system. This implies on the one hand that there is no local order parameter through which such phases can be distinguished from a trivial one, leading to necessary theoretical refinements of Landau's ideas. On the other hand, topological phases have physical properties (such as their metallic boundary states or quantized responses to external fields) protected from local perturbations (such as defects, impurities or other materical imperfections). In fact, their responses typically depend on combinations of fundamental constants (like e,h,c) and not on microscopic details - like the Hall conductivity which is an integer multiple of e^2/h. These features make the study of topological phases of course very exciting, and the topic has become one of the hottest areas of condensed matter physics and beyond.

The purpose of this class is to give an elementary introduction to the topic, based on simple examples and with many explicit calculations. The only prerequisite is a solid knowledge of the basic principles of quantum mechanics and matrix algebra.



The ground state of the AKLT model as an MPS state

Scheduling:

The class will follow an intense schedule with meetings on Tue and Thu from 4 to 6.50 pm. in GFS108. First class will be on Jan 31st. For further information contact Prof. H. Saleur: saleur@usc.edu

720: INTRODUCTION TO TOPOLOGICAL PHASES IN CONDENSED MATTER

By Pr. H. Saleur

Syllabus

Chapters 1–7 deal mostly with non-interacting systems (in different dimensions). Chapter 8 allows for interactions but is restricted to one dimension.

CHAPTER 1: REMINDERS AND GENERALITIES

- 0: 1D tight-binding chains
- 1: Reminders on first and second quantization
- 2: SSH and Rice-Mele model

CHAPTER 2: SYMMETRIES

- 0: Unitary symmetries
- 1: Non-unitary symmetries:
  - a: Time-reversal symmetry
  - b: Particle-hole symmetry
  - c: Chiral symmetry
  - d: The 10-fold way

CHAPTER 3: PHENOMENOLOGY OF TOPOLOGICAL PHASES FROM 1D EXAMPLES

- 1: Topological properties of the SSH model
- 2: Polarization, Berry and Zak phase
- 3: The invariant from inversion eigenvalues
- 4: Edge states
- 5: Edge states from the Dirac equation
- 6: Wannier centers and charge pumping
- 7: The SSH model and the 10-fold way
- 8: 2D insulators and obstructed insulators
- 9: The Kitaev wire

CHAPTER 4: CHERN INSULATORS AND TOPOLOGICAL PHASES IN 2D

- 1: Chern insulators and the Chern number
- 2: Chern number and the Hal conductivity
- 3: Bulk-boundary correspondence. Chiral boundary modes
- 4: 2D quantum Spin-Hall insulators

CHAPTER 5: 3D TOPOLOGICAL INSULATORS IN CLASS AIII

CHAPTER 6: WEYL SEMIMETALS

CHAPTER 7: THE ONE-DIMENSIONAL CASE

- 0: Entanglement
- 1: The AKLT spin chain - basics
- 2: Non-local order parameter in AKLT
- 3: Matrix Product states
- 4: Projective group representations
- 5: SPT phases

BIBLIOGRAPHY

There is no textbook for this class. Research and review articles will be recommended for each chapter. Copies of my notes will be available.

GRADING

Homeworks will regularly be handed out. The final grade will be determined as a combination of the homeworks and a take home exam.