

Math 533

Algebraic Combinatorics

3.0 Units Spring 2023 MWF 1:00-1:50pm

Location: KAP 148

Instructor: Sami Assaf (shassaf@usc.edu)

Office: KAP 438-C, Zoom PMI 539 730 6919

Office Hrs: Mondays and Fridays at 2pm

Course Description

This course looks in depth at topics in algebraic combinatorics, with emphasis on combinatorial objects such as Young tableaux, pipe dreams, puzzles, and Kohnert diagrams which generate Schur and Schubert polynomials. We will explore the geometric context for these polynomials and the associated combinatorics, though no prior course in Algebraic Geometry is required.

Learning Objectives

Students will develop proficiency implementing combinatorial algorithms on tableaux and recognizing trends among generalizations thereof; recognize how tableaux combinatorics can resolve positivity problems in symmetric polynomials and generalizations thereof; articulate the geometric context underlying the classical tableaux combinatorics for Grassmannians.

Prerequisite(s): MATH 410 and MATH 532

Required Readings and Supplementary Materials

Required: *Young tableaux* by William Fulton.

Description and Assessment of Assignments

Students are expected to complete two projects during the term. The first project is a 50 minute group presentation of one of three classic papers generalizing the Robinson-Schensted algorithm. Presentations will take place during the eighth week of class, on Feb 27, Mar 1, and Mar 3. The second project is a written report on a specific topic in Schubert calculus, options TBD. This will fulfill the final exam requirement for the course, due on Wednesday May 3 by 4pm.

Homework problems will be assigned throughout the term. Completion of homework is optional.

Grading Breakdown

Assignment	% of Grade
Oral Presentation	50%
Final Exam (05/03)	50%
TOTAL	100%

Course Schedule

	Topics/Daily Activities	Readings
Weeks 1-7	Tableaux Calculus	Young Tableaux Part I
Week 8	Oral Presentations	
Weeks 9-15	Schubert Calculus	Young Tableaux Part III

Week	Monday	Wednesday	Friday
1	01/09 Notation (pg 1-4)	01/11 1.1 Row-insertion	01/13 1.2 Sliding: jeu de taquin
2	01/16 Martin Luther King Jr Day	01/18 2.1 Words and transformations	01/20 2.2 Schur polynomials
3	01/23 2.3 Column Words	01/25 3 Increasing sequences	01/27 Robinson--Schensted--Knuth
4	01/30 Presentations Overview	02/01 4.1 The correspondence	02/03 4.2 Matrix-ball construction
5	02/06 4.3 Applications of RSK	02/08 5.1 Skew tableaux	02/10 5.2 Reverse lattice words
6	02/13 5.3 Littlewood--Richardson	02/15 6.1 Symmetric polynomials	02/17 6.2 Ring of symmetric functions
7	02/20 Presidents' Day	02/22 Robinson's proof of LRR	02/24 Dual equivalence
8	02/27 Group Presentations	03/01 Group Presentations	03/03 Group Presentations
9	03/06 Stanley symmetric functions	03/08 Slide polynomials	03/10 Schubert polynomials
	03/13	03/15	03/17
	Spring Break		
10	03/20 Grassmannian Varieties	03/22 9.1 Plücker coordinates	03/24 9.1 Projective Embeddings
11	03/27 9.2 Invariant theory	03/29 9.3 Line bundles	03/31 9.4 Grassmannians
12	04/03 9.4 Grassmannians	04/05 10.1 Torus fixed points	04/07 10.2 Schubert varieties
13	04/10 10.2 Schubert classes	04/12 10.3 Relations of Schuberts	04/14 10.5 Bruhat order
14	04/17 10.4 Schubert polynomials	04/19 Properties of Schubert polys	04/21 10.6 Schur polynomials
15	04/24 Pipe dreams	04/26 Kohnert's rule	04/28 Schubert times Schur coeffs

Statement on Academic Conduct and Support Systems

Academic Conduct:

Plagiarism – presenting someone else’s ideas as your own, either verbatim or recast in your own words – is a serious academic offense with serious consequences. Please familiarize yourself with the discussion of plagiarism in SCampus in Part B, Section 11, “Behavior Violating University Standards” policy.usc.edu/scampus-part-b. Other forms of academic dishonesty are equally unacceptable. See additional information in SCampus and university policies on [Research and Scholarship Misconduct](#).

Students and Disability Accommodations:

USC welcomes students with disabilities into all of the University’s educational programs. The Office of Student Accessibility Services (OSAS) is responsible for the determination of appropriate accommodations for students who encounter disability-related barriers. Once a student has completed the OSAS process (registration, initial appointment, and submitted documentation) and accommodations are determined to be reasonable and appropriate, a Letter of Accommodation (LOA) will be available to generate for each course. The LOA must be given to each course instructor by the student and followed up with a discussion. This should be done as early in the semester as possible as accommodations are not retroactive. More information can be found at osas.usc.edu. You may contact OSAS at (213) 740-0776 or via email at osasfrontdesk@usc.edu.

Support Systems:

Counseling and Mental Health - (213) 740-9355 – 24/7 on call
studenthealth.usc.edu/counseling

Free and confidential mental health treatment for students, including short-term psychotherapy, group counseling, stress fitness workshops, and crisis intervention.

National Suicide Prevention Lifeline - 1 (800) 273-8255 – 24/7 on call
suicidepreventionlifeline.org

Free and confidential emotional support to people in suicidal crisis or emotional distress 24 hours a day, 7 days a week.

Relationship and Sexual Violence Prevention Services (RSVP) - (213) 740-9355(WELL), press “0” after hours – 24/7 on call
studenthealth.usc.edu/sexual-assault

Free and confidential therapy services, workshops, and training for situations related to gender-based harm.

Office for Equity, Equal Opportunity, and Title IX (EEO-TIX) - (213) 740-5086
eetix.usc.edu

Information about how to get help or help someone affected by harassment or discrimination, rights of protected classes, reporting options, and additional resources for students, faculty, staff, visitors, and applicants.

Reporting Incidents of Bias or Harassment - (213) 740-5086 or (213) 821-8298
usc-advocate.symplicity.com/care_report

Avenue to report incidents of bias, hate crimes, and microaggressions to the Office for Equity, Equal Opportunity, and Title for appropriate investigation, supportive measures, and response.

The Office of Student Accessibility Services (OSAS) - (213) 740-0776
osas.usc.edu

OSAS ensures equal access for students with disabilities through providing academic accommodations and auxiliary aids in accordance with federal laws and university policy.

USC Campus Support and Intervention - (213) 821-4710

campussupport.usc.edu

Assists students and families in resolving complex personal, financial, and academic issues adversely affecting their success as a student.

Diversity, Equity and Inclusion - (213) 740-2101

diversity.usc.edu

Information on events, programs and training, the Provost's Diversity and Inclusion Council, Diversity Liaisons for each academic school, chronology, participation, and various resources for students.

USC Emergency - UPC: (213) 740-4321, HSC: (323) 442-1000 – 24/7 on call

dps.usc.edu, emergency.usc.edu

Emergency assistance and avenue to report a crime. Latest updates regarding safety, including ways in which instruction will be continued if an officially declared emergency makes travel to campus infeasible.

USC Department of Public Safety - UPC: (213) 740-6000, HSC: (323) 442-120 – 24/7 on call

dps.usc.edu

Non-emergency assistance or information.

Office of the Ombuds - (213) 821-9556 (UPC) / (323-442-0382 (HSC)

ombuds.usc.edu

A safe and confidential place to share your USC-related issues with a University Ombuds who will work with you to explore options or paths to manage your concern.

Occupational Therapy Faculty Practice - (323) 442-3340 or otfp@med.usc.edu

chan.usc.edu/otfp

Confidential Lifestyle Redesign services for USC students to support health promoting habits and routines that enhance quality of life and academic performance.