

**MATH 599 TOPICS IN COMBINATORICS:
DISCRETE GEOMETRY AND STATISTICAL MECHANICS
FALL 2022 SYLLABUS**

GRETA PANOVA

Location and meeting times: 3 academic hours per week distributed as two 1.5 hour lectures. Location and time TBA.

Course description. This course will consist of two parts, which are not directly related.

The first part will study convex discrete geometry. We will start with general notions of convexity, point configurations, Helly's theorem, Ham sandwich theorem etc. We'll cover Minkowski's theorem for lattices. We will study point configurations and Ramsey-type theorems on convex independent sets and point-line incidences. Next, we will consider polyhedra and polytopes, and we will explore their enumerative properties – counting integer points, Ehrhart theory, f and h -vectors, reciprocity theorems etc.

The second and largely unrelated topic is about statistical mechanics, and in particular – dimer configurations in 2 dimensions. Such configurations include lozenge tilings on the hexagonal grid, dimer covers on the square grid etc. We will study their height functions, partitions functions, Kasteleyn theory, and discuss probabilistic/limiting behavior of such models.

The unifying theme in these topics comes to light in the limit case, understanding the asymptotic behavior and typical configurations. At the heart of both topics lie the methods of generating functions and their generalizations.

Prerequisites. Combinatorics (Math 532 or equivalent), Linear and Abstract Algebra (510 or equivalent); or explicit permission from the instructor.

Textbooks and sources.

[JM] Jiri Matousek, *Lectures on Discrete Geometry*, 2002.

[RS] Richard Stanley, *Enumerative combinatorics, volume 1*, 2015.

[MBRS] Matthias Beck and Raman Sanyal, *Combinatorial reciprocity theorems*, 2018. <https://matthbeck.github.io/papers/crt.pdf>

[RK] Richard Kenyon, *Lectures on dimers*, <https://arxiv.org/pdf/0910.3129.pdf>.

[VG] Vadim Gorin, *Lectures on random lozenge tilings*, https://people.math.wisc.edu/~vadicgor/Random_tilings.pdf.

Course notes and materials. All notes and other materials will be uploaded on the course blackboard website (assigned by USC).

Assessment.

Assessment	Points	Grade %
Homework 1	15	15%
Homework 2	15	15%
Homework 3	15	15%
Homework 4	15	15%
Homework 5	15	15%
Presentation	25	25%

Grading.

The grade will be determined by the table above. Homeworks will be submitted on the blackboard page and graded within a week. The presentation will consist of a 50 minute lecture on a pre approved topic of interest and will be scheduled for each student individually throughout the semester. The final score will be determined by the formula above. Letter grades will be assigned on a curve, i.e. relative to everyone's performance, with mean/median centered at A-.

Weekly schedule. The daily course schedule including contents of each lecture, and homework assignments will be regularly posted on the blackboard course website.

Week	Topic	Reading	Deliverables
Week 1	Convexity. Caratheodory and Helly's theorem, ham sandwich.	[JM] Chapter 1	
Week 2	Lattices. Minkowski's theorem. The Erdős-Szekeres theorem.	[JM] Chapter 2 and 3	
Week 3	Incidence problems. Szemerédi–Trotter.	[JM] Chapter 4	Homework 1 due
Week 4	Convex polytopes. H- and V-polytopes. Faces.	[JM] Chapter 5	
Week 5	Enumeration, the Ehrhart polynomial.	[MBRS] Chapter 1	
Week 6	Partially ordered sets, two poset polytopes, zeta polynomials, Möbius function.	[MBRS] Chapter 2	Homework 2 due
Week 7	The face lattice and Zaslavsky's theorem.	[MBRS] Chapter 3	
Week 8	Rational generating functions.	[MBRS] Chapter 4	
Week 9	Subdivisions and reciprocity. The h^* - and f - vectors. Dehn-Sommerville relations.	[MBRS] Chapter 5	Homework 3 due
Week 10	Hyperplane arrangements. Zonotopes and tilings.	[MBRS] Chapter 7	
Week 11	Dimers and random surfaces, overview.	[RK] Section 1	
Week 12	The height functions. Plane partitions, revisited.	[RK] Section 2 and [VG] Chapter 1	Homework 4 due
Week 13	Kasteleyn theory.	[RK] Section 3	
Week 14	Partition functions.	[RK] Section 4	
Week 15	Lozenge tilings and algebraic combinatorics.	[VG] Section 3	
Week 16	Limiting behavior investigations.	[RK] Section 9 and 10	Homework 5 due

STATEMENT OF ACADEMIC CONDUCT AND SUPPORT SYSTEMS

Academic Conduct:

Plagiarism – presenting someone else’s ideas as your own, either verbatim or recast in your own words – is a serious academic offense with serious consequences. Please familiarize yourself with the discussion of plagiarism in SCampus in Part B, Section 11, “Behavior Violating University Standards” policy.usc.edu/scampus-part-b. Other forms of academic dishonesty are equally unacceptable. See additional information in SCampus and university policies on Research and Scholarship Misconduct.

Students and Disability Accommodations:

USC welcomes students with disabilities into all of the University’s educational programs. The Office of Student Accessibility Services (OSAS) is responsible for the determination of appropriate accommodations for students who encounter disability-related barriers. Once a student has completed the OSAS process (registration, initial appointment, and submitted documentation) and accommodations are determined to be reasonable and appropriate, a Letter of Accommodation (LOA) will be available to generate for each course. The LOA must be given to each course instructor by the student and followed up with a discussion. This should be done as early in the semester as possible as accommodations are not retroactive. More information can be found at osas.usc.edu. You may contact OSAS at (213) 740-0776 or via email at osasfrontdesk@usc.edu.

Support Systems:

Counseling and Mental Health - (213) 740-9355 – 24/7 on call studenthealth.usc.edu/counseling Free and confidential mental health treatment for students, including short-term psychotherapy, group counseling, stress fitness workshops, and crisis intervention.

National Suicide Prevention Lifeline - 1 (800) 273-8255 – 24/7 on call suicidepreventionlifeline.org Free and confidential emotional support to people in suicidal crisis or emotional distress 24 hours a day, 7 days a week.

Relationship and Sexual Violence Prevention Services (RSVP) - (213) 740-9355(WELL), press “0” after hours – 24/7 on call studenthealth.usc.edu/sexual-assault Free and confidential therapy services, workshops, and training for situations related to gender-based harm.

Office for Equity, Equal Opportunity, and Title IX (EEO-TIX) - (213) 740-5086 eeotix.usc.edu Information about how to get help or help someone affected by harassment or discrimination, rights of protected classes, reporting options, and additional resources for students, faculty, staff, visitors, and applicants.

Reporting Incidents of Bias or Harassment - (213) 740-5086 or (213) 821-8298 usc-advocate.symplicity.com/care_report Avenue to report incidents of bias, hate crimes, and microaggressions to the Office for Equity, Equal Opportunity, and Title for appropriate investigation, supportive measures, and response.

The Office of Student Accessibility Services (OSAS) - (213) 740-0776 osas.usc.edu OSAS ensures equal access for students with disabilities through providing academic accommodations and auxiliary aids in accordance with federal laws and university policy. USC Campus Support and Intervention - (213) 821-4710 campussupport.usc.edu Assists students and families in resolving complex personal, financial, and academic issues adversely affecting their success as a student.

Diversity, Equity and Inclusion - (213) 740-2101 diversity.usc.edu Information on events, programs and training, the Provost's Diversity and Inclusion Council, Diversity Liaisons for each academic school, chronology, participation, and various resources for students.

USC Emergency - UPC: (213) 740-4321, HSC: (323) 442-1000 – 24/7 on call dps.usc.edu, emergency.usc.edu Emergency assistance and avenue to report a crime. Latest updates regarding safety, including ways in which instruction will be continued if an officially declared emergency makes travel to campus infeasible.

USC Department of Public Safety - UPC: (213) 740-6000, HSC: (323) 442-120 – 24/7 on call dps.usc.edu Non-emergency assistance or information.

Office of the Ombuds - (213) 821-9556 (UPC) / (323-442-0382 (HSC) ombuds.usc.edu A safe and confidential place to share your USC-related issues with a University Ombuds who will work with you to explore options or paths to manage your concern.

Occupational Therapy Faculty Practice - (323) 442-3340 or otfp@med.usc.edu chan.usc.edu/otfp Confidential Lifestyle Redesign services for USC students to support health promoting habits and routines that enhance quality of life and academic performance.