EE 604: Computational Methods in Applied Physics

USC Viterbi School of Engineering
4 Units | Mon/Wed, 2:00–3:50pm | Location: KDC 241 | Fall 2022

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Course Webpage: https://blackboard.usc.edu

Course Description

Most physical models are formulated through differential equations that cannot be solved analytically except in special and simplified scenarios. This course covers methods for numerically solving complex linear and nonlinear differential equations and stochastic differential equations.

Many examples (primarily from photonics) are used to illustrate these numerical methods. For example, as examples for eigenmode computations, we will consider Anderson localization, distributed Bragg reflector (DBR), Fabry-Perot resonators, photonic band structure, whispering-gallery modes, and waveguide modes. As examples for linear response problems and spectral methods, we will consider resonant scattering and metasurfaces. As examples for nonlinear propagation and splitting methods, we will consider solitons and modulation instability.

The numerical methods covered include finite-difference discretization, eigen problems, linear response problems, perfectly matched layers, spectral methods, equivalent sources, near-to-far-field transformations, splitting methods for nonlinear propagation, Runge–Kutta methods, and stochastic integration.

This is a hands-on class where the class times are split between lectures and code writing. We will write all of the simulations ourselves. This way, students get to know how these numerical methods work and will be able to write their own simulations for custom problems.

The assignments consist of three mini-projects and one final project.

Slides summarizing this course: https://sites.usc.edu/hsugroup/files/2022/11/EE604.pptx

Learning Objectives

Upon successful completion of this course a student will be able to

- Translate eigen problems of complex systems into matrix equations for numerical solution.
- Solve linear response problems with appropriate sources and boundary conditions.
- Simulate linear and nonlinear wave propagation.
- Numerically solve stochastic differential equations.

Prerequisites: None.

Required preparation: Familiarity with (1) differential equations and wave equations at the level of EE 370 or beyond, (2) computer programming experience at the level of EE 155L or beyond, (3) Fourier transforms.

Required Software: MATLAB, available to USC students at https://software.usc.edu/matlab/

Primary Course Materials: Lecture notes and codes prepared by instructor, to be posted on Blackboard.

Supplementary Course Materials:

Supplementary (not required) materials are taken from the following textbooks:

1) John D. Joannopoulos, Steven G. Johnson, Joshua N. Winn, and Robert D. Meade, *Photonic Crystals: Molding the Flow of Light* (2nd edition), Princeton University Press (2008); ISBN 978-0691124568; freely available online at http://ab-initio.mit.edu/book/photonic-crystals-book.pdf
2) Allen Taflove & Susan C. Hagness, *Computational Electrodynamics: The Finite-Difference Time-Domain Method* (3rd Edition), Artech House (2005); ISBN 978-1580538329.
3) Jian-Ming Jin, *Theory and Computation of Electromagnetic Fields* (2nd edition), Wiley (2015); ISBN 978-1119108047; freely available with USC account at https://ebookcentral.proquest.com/lib/socal/detail.action?docID=4043026&pq-origsite=primo
4) Peter J. Olver, *Introduction to Partial Differential Equations*, Springer (2014) (2016 printing with typo corrections); ISBN 978-3319020983; freely available online within USC internet at https://link-springer-com.libproxy1.usc.edu/book/10.1007/978-3-319-02099-0
5) Govind Agrawal, *Nonlinear Fiber Optics* (6th edition), Elsevier Academic Press (2019); ISBN 978-0128170427; freely available online within USC internet at https://www-sciencedirect-com.libproxy2.usc.edu/book/9780128170427/nonlinear-fiber-optics
6) J. C. Butcher, *Numerical Methods for Ordinary Differential Equations* (3rd edition), Wiley (2016); ISBN 978-1119121534; freely available with USC account at https://ebookcentral.proquest.com/lib/socal/detail.action?docID=4591869&pq-origsite=primo
7) Peter E. Kloeden & Eckhard Platen, *Numerical Solution of Stochastic Differential Equations*, Springer (1992); ISBN 978-3662126165; freely available online within USC internet at https://link-springer-com.libproxy2.usc.edu/book/10.1007%2F978-3-662-12616-5

Grading Breakdown

20% Mini-project 1
20% Mini-project 2
20% Mini-project 3
20% Final project presentation
20% Final project write-up

Assignment Description

- There will be **3 mini-projects**:
  1. Band-pass filter using resonant transmission
  2. Flat lens using metasurface
  3. Fiber optical communication with solitons

  In each mini-project, the student will be guided through multiple parts that involve writing codes to simulate the problem, coming up with the design and validating it numerically, and comparing different approaches — utilizing the computational methods taught in the preceding weeks of the course.

The mini-projects will be graded based on (1) completeness and readability of the codes, (2) whether the methods used are appropriate, and (3) correctness of the answers. Students will need to submit their codes, plots, and accompanying texts.

Codes developed in class (which will be posted on Blackboard) can be freely used for completion of the assignments. Those codes will be in MATLAB, so the use of MATLAB will make the assignments easier, but students are allowed to use any programming language and any built-in function or package. **Codes must not be shared among students.**

- There will be **one final project**, for which each student chooses a topic with guidance/feedback from the instructor. A suitable project should involve using or extending the numerical methods covered in this class, and applying them onto a problem of the student's interest. **The amount of work should roughly equal that of two mini-projects**, but in a more exploratory way.

  An example final project from the past is "Frequency comb generation from the Lugiato–Lefever equation," which uses the split-step Fourier method covered in this class but generalizes to the Lugiato–Lefever equation and apply it for frequency comb generation. Another example final project from the past is "Guides resonances and bound states in the continuum in dielectric gratings" which utilizes a frequency-domain eigensolver with Bloch periodic boundary condition and perfectly-matched layers to compute the modes of interest.

  Each student will (1) give a **10-to-12-minute presentation to the class**, and (2) write a **2-to-4-page report** in the style of a single-spaced double-column journal paper. The presentations and report should have the fellow classmate as the target audience.

  Final project time line:
  - Weeks 9-10: Discuss final project topic with the instructor.
  - **Friday 10/28: Final project topic due**.
  - **Mon 11/28 & Wed 11/30: Final project presentation**.
  - **Friday Dec 9 at 4pm: final project write-up due on Blackboard**.

  The final project will be graded based on (1) whether the topic and the amount of work is appropriate, (2) completeness of the study, and (3) clarity and effectiveness of the presentation, write-up, and codes. Novelty/originality is a bonus, but is not required (eg: reproducing results from a published paper is appropriate, but the paper should be a recent one).

Assignment Submission Policy

- All assignment will be submitted on Blackboard, as code files and PDFs.
- Late Policy: Late assignment will not be accepted except for institution-established emergency reasons. Credit for such late assignment is with the discretion of the instructor.

Course Schedule:

Date	Topics	Deliverables
Week 1 8/22, 8/24	**Eigenmodes of closed systems in 1D** Mon: Course info & objectives. Wave equations. Finite-difference schemes. Matrix representation of differential operators. Boundary condition. Frequency-domain eigen problems. Anderson localization in 1D. Wed: Numerical dispersion. Bloch periodic boundary condition. Distributed Bragg reflector (DBR). Generalized eigen problem. [Supplemental: Joannopoulos p49-52].	
Week 2 8/29, 8/31	**Open systems: resonances & scattering in 1D** Mon: Open systems. Resonant/leaky modes. Outgoing boundary condition. Nonlinear eigenvalue problem. Wed: Point-source response (Green's function). Scattering problems. Volume source. Transmission through a Fabry-Perot etalon.	Mini-project 1 posted: Band-pass filter using resonant transmission
Week 3 9/7	**Eigenmodes of closed systems in 2D** *No class on 9/5 (Labor Day).* Wed: Operators in 2D and Kronecker product. Eigenmodes and Anderson localization in 2D. Sparse matrices. Band structure of 2D photonic crystals. [Supplemental: Joannopoulos p66-69].	
Week 4 9/12, 9/14	**Open systems in 2D and PML** Mon: Propagating & evanescent waves. Perfectly matched layer (PML) through analytic continuation onto complex plane. Wed: PML reflection in continuous limit and from discretization. Real and imaginary coordinate stretching profiles. PML discretization. [Supplemental: Taflove Chapter 7]	9/16: Mini-project 1 due
Week 5 9/19, 9/21	**PML implementation & Metasurfaces** Mon: PML Implementation and validation in 1D and 2D. Point-source response in a 2D open system. Wed: Metasurfaces. Angular spectrum propagation (ASP). Locally periodic approximation.	
Week 6 9/26, 9/28	**Free-space propagation & FFT & waveguide modes** Mon: Discrete-time Fourier transform and discrete Fourier transform. Nyquist–Shannon sampling criterion & aliasing. Wed: ASP implementation. Waveguide modes: band structure and its computation as eigen problems.	Mini-project 2 posted: Flat lens using metasurface
Week 7 10/3, 10/5	**Whispering-gallery modes & flux & surface source** Mon: Waveguide mode computation. Whispering-gallery modes and their excitation. Wed: Mode conversion. Flux computation. Surface source.	
Week 8 10/10, 10/12	**Scattering cross sections & symmetries** Mon: Transport and scattering through a waveguide bent. Scattering & absorption & extinction cross sections. Wed: Symmetry operators and how to exploit symmetries.	10/10: Mini-project 2 due.
Week 9 10/17, 10/19	**Near-to-far-field transformation & integral-equation methods** Mon: Near-to-far-field transformation (N2FFT): derivation and implementation. Differential scattering cross section. [Supplemental: Taflove Chapter 8]	Discuss final project with instructor.

	Wed: Integral equation methods with Green's function. Cherenkov radiation.	
Week 10 10/24, 10/26	**Far-field limit & optical theorem & subpixel smoothing** Mon: N2FFT in the far field. Fraunhofer distance. Scattering amplitude. Wed: Optical theorem in 2D and 3D. Subpixel smoothing.	10/28: Final project topic due.
Week 11 10/31, 11/2	**Nonlinear wave propagation in 1+1D** Mon: Nonlinear Schrödinger equation (NLSE) in space and in pulse propagation. Diffraction/dispersion length and nonlinear length. Solitons. Wed: Split-step Fourier method (SSFM) and Strang splitting. Supplemental reading: Agrawal chapter 2	
Week 12 11/7, 11/9	**Splitting methods & Nonlinear wave propagation in 1+2D** Mon: SSFM implementation. Numerical experiments: fundamental & higher-order solitons, soliton attractions & repulsions. Modulation instability. Wed: NLSE and SSFM in 1+2 dimensions. Wave function collapse. Soliton in 1+2D.	Mini-project 3 posted: Fiber optical communication with solitons
Week 13 11/14, 11/16	**Boundary value problem & integration of ordinary differential equations** Mon: Shooting method for boundary value problem. Bisection method for root finding. Numerical solution of soliton in 1+2D. Wed: Runge–Kutta methods: second-order, fourth-order, and adaptive Runge–Kutta smethods. Supplemental reading: Butcher chapter 2 section 23.	11/14: iMini-project 3 due
Week 14 11/21	**Inverse design & adjoint method** Mon: gradient computation using adjoint method. Derivative of matrix inverse. Nonlinear optimization. Inverse design of a waveguide bend. Supplemental reading: Steven Johnson's note on adjoint: https://github.com/mitmath/18335/blob/spring21/notes/adjoint/adjoint.pdf Wed: *No class (Thanksgiving)*	
Week 15 11/28, 11/30	**Final project presentation**	Final project presentation.
Final Week	Final project write-up due on Blackboard at 4pm on Friday Dec 9th.	Final project write-up due