MATH 606, Summer 2022.
Topics in Stochastic Processes (054--39482R)
Gaussian Processes

Class meetings: MW, 9:30am-12:30pm, VHE 210.

Information on this and related pages changes frequently.

Instructor: Sergey Lototsky.
Office: KAP 248D.
Phone: 213 740 2389
E-mail: lototsky usc edu.

URL: https://dornsife.usc.edu/sergey-lototsky/
https://dornsife.usc.edu/sergey-lototsky/math-606-summer-2022/

Office Hours: MW before and after the class. Appointments at other time are welcome.

Course objective: To learn the foundations of the theory of Gaussian processes. More specifically, a Gaussian process $X = X(t)$, $t \in [0, T]$, is a collection of random variables such that, for every finite set $\{t_1, \ldots, t_n\} \subset [0, T]$, the random vector $(X(t_1), \ldots, X(t_n))$ is Gaussian. Such a process has a number of remarkable properties. The story becomes even more interesting once we allow the domain of $X$ to be an arbitrary set and allow $X$ to take values in a locally convex linear topological space. In this class, we will use probabilistic and analytical tools to understand basic results in the theory of Gaussian processes. The topics will include:

- Main examples [Brownian motion, bridge, and sheet; Ornstein-Uhlenbeck process, fractional Brownian motion, Gaussian free field, etc.]
- Various representations of the Gaussian processes;
- Basic properties of sample paths (continuity, Borel-TIS inequality, large and small deviations, etc.);
- Spectral theory;
- Gaussian measures on a locally convex linear topological space;
- Cameron-Martin theorem.

Course work: Class participation, homework assignments, final presentation.

Official grading scheme: 20% class participation, 40% homework assignments, 40% final presentation.

**Other references**

- Adler, Robert J. An introduction to continuity, extrema, and related topics for general Gaussian processes. *Institute of Mathematical Statistics, Hayward, CA*, 1990. **x+160 pp.**

• Kocijan, Juš Modelling and control of dynamic systems using Gaussian process models. *Springer, Cham*, 2016. **xvi+267 pp.**
• Shi, Jian Qing; Choi, Taeryon Gaussian process regression analysis for functional data. *CRC Press, Boca Raton, FL*, 2011. **xx+196 pp.**
• Rosenblatt, Murray Gaussian and non-Gaussian linear time series and random fields. *Springer-Verlag, New York*, 2000. **xiv+246 pp.**

• Ledoux, Michel The concentration of measure phenomenon. *American Mathematical Society, Providence, RI*, 2001. **x+181 pp.**

An example of a book review from *Math reviews* [Edition 1, Edition 2] and from *the Bulletin of the AMS*

**The course file**, including homework problems. **Aim at two problems per week.**

**My notes:**

• [A time line](#)
• [Gaussian objects](#)
• Pictures of Brownian sheet, The Kiefer field, Brownian bridge twice, GFF; the Matlab code
• [Mercer's theorem](#)
• [Abstract Wiener space](#)
• RKHS
• [Stochastic analysis in continuous time](#)
• [A summary of Brownian motion](#)
• [A summary of Gaussian inequalities](#)
• [A summary of large deviations](#)
• [A summary of SODEs](#)
• [A summary of the Cameron-Martin-Girsanov theorem and related results](#)
• [The Weierstrass Approximation Theorem](#)
• [Notes about Kalman filter and related topics](#)

**Other notes**

• Lecture notes for math 547 (statistical learning theory) by Steven Heilman
Our progress

May 18: Various characterizations of a Gaussian vector.

May 23: Examples of Gaussian processes.


May 30: Memorial Day, no class.

June 1: Abstract Wiener space.

June 6: RKHS; Markov property.

June 8: An overview of Gaussian inequalities.

June 13: Gaussian inequalities.

June 15: Large and small deviations.

June 20: Large deviations and applications.

June 22: Cameron-Martin-Girsanov theorem.

June 27: Filtering in general and Kalman filter in particular.

June 29: The final discussion.