MATH 606, Summer 2022.
Topics in Stochastic Processes (054--39482R)
Gaussian Processes
Class meetings: MW, 9:30am-12:30pm, VHE 210.

Information on this and related pages changes frequently.

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Office Hours: MW before and after the class. Appointments at other time are welcome.

Course objective: To learn the foundations of the theory of Gaussian processes. More specifically, a Gaussian process $X = X(t)$, $t \in [0, T]$, is a collection of random variables such that, for every finite set $\{t_1, \ldots, t_n\} \subset [0, T]$, the random vector $(X(t_1), \ldots, X(t_n))$ is Gaussian. Such a process has a number of remarkable properties. The story becomes even more interesting once we allow the domain of X to be an arbitrary set and allow X to take values in a locally convex linear topological space. In this class, we will use probabilistic and analytical tools to understand basic results in the theory of Gaussian processes. The topics will include

- Main examples [Brownian motion, bridge, and sheet; Ornstein-Uhlenbeck process, fractional Brownian motion, Gaussian free field, etc.]
- Various representations of the Gaussian processes;
- Basic properties of sample paths (continuity, Borel-TIS inequality, large and small deviations, etc.);
- Spectral theory;
- Gaussian measures on a locally convex linear topological space;
- Cameron-Martin theorem.

Course work: Class participation, homework assignments, final presentation.

Official grading scheme: 20% class participation, 40% homework assignments, 40% final presentation.

Main reference: Mikhail Lifshits, Lectures on Gaussian processes. Springer Briefs in Mathematics. Springer, Heidelberg, 2012. **x+121 pp**. The book is available in electronic form from the USC Libraries

Other references

- Mishura, Yuliya; Zili, Mounir Stochastic analysis of mixed fractional Gaussian processes. *ISTE Press, London; Elsevier Ltd, Oxford,* **2018. xvi+194 pp.**
- Bovier, Anton Gaussian processes on trees: From spin glasses to branching Brownian motion. *Cambridge University Press, Cambridge,* 2017. **x+200 pp.**
- Adler, Robert J. The geometry of random fields. Reprint of the 1981 original. *SIAM, Philadelphia, PA,* 2010. **xxi+280 pp.**
- Adler, Robert J.; Taylor, Jonathan E. Random fields and geometry.*Springer, New York,* 2007. **xviii+448 pp.**
- Marcus, Michael B.; Rosen, Jay Markov processes, Gaussian processes, and local times. *Cambridge University Press, Cambridge,* 2006. **x+620 pp.**
- Piterbarg, Vladimir I. Asymptotic methods in the theory of Gaussian processes and fields. Translated from the Russian by V. V. Piterbarg. Revised by the author. *American Mathematical Society, Providence, RI,* 1996. **xii+206 pp.**
- Lifshits, M. A. Gaussian random functions. *Kluwer Academic Publishers, Dordrecht,* 1995. **xii+333 pp.**
- Yurinsky, Vadim Sums and Gaussian vectors. Lecture Notes in Mathematics, 1617. *Springer-Verlag, Berlin,* 1995. **xii+305 pp.**
- Hida, Takeyuki; Hitsuda, Masuyuki Gaussian processes. *American Mathematical Society, Providence, RI,* 1993. **xvi+183 pp.**
- Adler, Robert J. An introduction to continuity, extrema, and related topics for general Gaussian processes. *Institute of Mathematical Statistics, Hayward, CA,* 1990. **x+160 pp.**
- Ibragimov, Il′dar Abdullovich; Rozanov, Y. A. Gaussian random processes. *Springer-Verlag, New York-Berlin,* 1978. **x+275 pp.**
- Dym, H.; McKean, H. P. Gaussian processes, function theory, and the inverse spectral problem. *Academic Press, New York-London,* 1976. **xi+335 pp.**

- Urbina-Romero, Wilfredo Gaussian harmonic analysis. *Springer, Cham,* 2019. **xix+477 pp.**
- Hu, Yaozhong Analysis on Gaussian spaces. *World Scientific,* 2017. **xi+470 pp.**
- Mandrekar, Vidyadhar S.; Gawarecki, Leszek Stochastic analysis for Gaussian random processes and fields. *CRC Press, Boca Raton, FL,* 2016. **xxii+179 pp.**
- Bogachev, Vladimir I. Gaussian measures. *American Mathematical Society, Providence, RI,* 1998. **xii+433 pp.**
- Janson, Svante Gaussian Hilbert spaces. *Cambridge University Press, Cambridge,* 1997. **x+340 pp.**
- Kuo, Hui Hsiung Gaussian measures in Banach spaces. Lecture Notes in Mathematics, Vol. 463. *Springer-Verlag, Berlin-New York,* 1975. **vi+224 pp.**

- Kocijan, Juš Modelling and control of dynamic systems using Gaussian process models. *Springer, Cham,* 2016. **xvi+267 pp.**
- Gualtierotti, Antonio F. Detection of random signals in dependent Gaussian noise. *Springer, Cham,* 2015. **xxxiv+1176 pp.**
- Shi, Jian Qing; Choi, Taeryon Gaussian process regression analysis for functional data. *CRC Press, Boca Raton, FL,* 2011. **xx+196 pp.**
- Rasmussen, Carl Edward; Williams, Christopher K. I. Gaussian processes for machine learning. *MIT Press, Cambridge, MA,* 2006. **xviii+248 pp.**
- Rue, Håvard; Held, Leonhard Gaussian Markov random fields. *Chapman & Hall/CRC, Boca Raton, FL,* 2005. **xii+263 pp.**
- Rosenblatt, Murray Gaussian and non-Gaussian linear time series and random fields. *Springer-Verlag, New York,* 2000. **xiv+246 pp.**

- Talagrand, Michel Upper and lower bounds for stochastic processes. Second edition. *Springer, Cham,* 2021. **xviii+726 pp.**
- Ledoux, Michel; Talagrand, Michel Probability in Banach spaces: Isoperimetry and processes. Reprint of the 1991 edition. *Springer-Verlag, Berlin,* 2011. **xii+480 pp.**
- Ledoux, Michel The concentration of measure phenomenon. *American Mathematical Society, Providence, RI,* 2001. **x+181 pp.**

An example of a book review from *Math reviews* [Edition 1, Edition 2] **and from** the Bulletin of the AMS

The course file, including homework problems. **Aim at two problems per week.**

My notes:

- A time line
- Gaussian objects
- Pictures of Brownian sheet, The Kiefer field, Brownian bridge twice, GFF; the Matlab code
- Mercer's theorem
- Abstract Wiener space
- RKHS
- Stochastic analysis in continuous time
- A summary of Brownian motion
- A summary of Gaussian inequalities
- A summary of large deviations

- A summary of SODEs
- A summary of the Cameron-Martin-Girsanov theorem and related results
- The Weierstrass Approximation Theorem
- Notes about Kalman filter and related topics

Other notes

- Lecture notes for math 547 (statistical learning theory) by Steven Heilman
- Donsker's Theorem [lecture notes by Davar Khoshnevisan]
- An article about sampling theorem (in signal processing)
- The original paper by James Mercer containing his theorem
- The original paper by C. Borell
- The Brunn-Minkowski inequality
- Geometric Gaussian inequalities
- A survey of Kalman filter
- In nature: GFF, Corner Growth (Iceland), Corner Growth (Armenia)

Our progress

May 18: Various characterizations of a Gaussian vector.

May 23: Examples of Gaussian processes.

May 25: Spectral and KL representations.

May 30: Memorial Day, no class.

June 1: Abstract Wiener space.

June 6: RKHS; Markov property.

June 8: An overview of Gaussian inequalities.

June 13: Gaussian inequalities.

June 15: Large and small deviations.

June 20: Large deviations and applications.

June 22: The Cameron-Martin-Girsanov theorem.

June 27: Filtering in general and Kalman filter in particular.

June 29: The final discussion.