USC University of Southern California

# GEOL 558: Inverse Theory in the Earth Sciences

## Description

**Units:** 3
**Term:** Spring 2022

**Lectures:** Wed 2:00-4:20 pm
**Location:** ZHS 118 (Zumberge Hall, first floor)

**Instructor:** Sylvain Barbot (sbarbot@usc.edu)
**Office Hours:** (upon appointment).

## Course Description

The course provides an introduction to inverse methods applicable to a wide range of scientific problems. The class will introduce the Bayesian description of inverse problems and various optimization methods based on a Hilbert space. In addition, the class will cover practical approaches to solve typical inverse problems. We will introduce important concepts, such as non-uniqueness, resolution, and model uncertainties.

## Learning Objectives

By the end of this course, the students will be introduced to a quantitative description of forward and inverse problems and practical approaches to solve them.

## Prerequisites:

Working knowledge of algebra, calculus, and statistics is useful.

## Communication

Regular communication will be conducted through Blackboard (https://blackboard.usc.edu).

## Lectures

From Wednesday, January 12$^{th}$ to Wednesday, April 13$^{th}$ 2022 there will be **13 lectures**. Spring recess is March 13-20. The lecture of Wednesday, January 12$^{th}$ is on zoom.

## Examinations

The grades will be based on attendance (25%), assignments (50%), and a final project (25%).

## Required Materials

Access to a Posix computer with scientific computing tools (Matlab, Python) is required.

## Assignments

Some work will be assigned weekly.

## Classroom norms

Student participation during lecture is strongly encouraged. Always feel free to ask questions and clarifications.

## Lecture content

Forward problems

- Discretization of integral equations
- Setup of forward and inverse problems

Elements of statistics

- Single-variate probability
- Probability density function
- Normal distribution (Log-normal distribution, Chi-square distribution with one degree of freedom, Chi-square distribution with k degrees of freedom, Exponential distribution, Pareto distribution)
- Cumulative density function
- Change of variable
- Characteristic function
- Sums of independent random variables
- Central limit theorem
- Maximum-likelihood estimation

Multi-variate probability

- Joint probabilities
- Independent variables
- Moments and covariance matrix
- Correlation coefficient
- N-dimensional normal distribution
- Conditional probability
- Marginal probability
- Change of variables
- Sums of independent random variables
- Products and ratios of independent random variables
- The Cauchy distribution

Inverse Theory

- Definitions
- Homogeneous probability distributions
- Jeffrey's parameters
- Homogeneous distribution on a sphere
- Conjunction of probabilities

Definition of the inverse problem

- Joint prior information
- Theoretical probability density function

- Conjunction of information
- Solutions of the inverse problem
- Normal distribution of observations and predictions
- Normal prior information

Linear forward model

- The design matrix
- Resolution operator
- Uniform prior information
- Regularization by smoothing

Optimization

- Norms
- Over-determined least-squares
- Under-determined least-squares
- Other minimizations with constraints
- Pseudo inverse
- Model resolution matrix
- Data resolution matrix
- Stability of inverse solutions
- Tikhonov regularization

Lecture content is subject to change without warning.

## Textbooks

Aster, R.C., Borchers, B. and Thurber, C.H., 2018. *Parameter estimation and inverse problems*. Elsevier.

Parker, R.L., 1994. *Geophysical inverse theory* (Vol. 1). Princeton university press.

Tarantola, A., 2005. *Inverse problem theory and methods for model parameter estimation*. Society for Industrial and Applied Mathematics.