INTRODUCTION TO QUANTUM FIELD THEORY - PHYS720

More than a subfield of physics such as relativity or electromagnetism, quantum field theory is a "tool and concepts" box. It has an extraordinary wide range of applications, reaching from statistical mechanics (critical phenomena, disordered systems, physics out of equilibrium) to condensed matter physics (solid state, soft matter) to the physics of quantum systems (cold atoms) to dynamics systems (turbulence, complex systems) to, of course, high-energy physics and pure mathematics. This universality comes in part from a set of techniques - perturbative expansions and Feynman diagrams, path integrals, functional integrals, renormalization and renormalization group - that are now indispensable to any physicist.

This course will provide a pedestrian introduction to the topic, with a focus on the statistical mechanics point of view (though relationships with particle physics will also be addressed). These two quotes give a good first flavor of the topic:

In The Renormalization Group method you take a structure you don't understand and convert it to another structure you don't understand. You keep doing it until you finally understand. (M. Berry)

The career of a young theoretical physicist consists of treating the harmonic oscillator in ever-increasing levels of abstraction. (S. Coleman)

Rules: Homeworks will regularly be handed out. Copies of my hand-written notes will be available, together with detailed references (such as research articles etc) to be posted on Blackboard. The final grade will be based on the homeworks (60%) and a take home final exam (40%).

SYLLABUS

1 Aspects of classical and quantum mechanics: reminders

Classical mechanics; Lagrangian, Hamiltonian. Quantum mechanics: quantization, Heisenberg and Schrödinger representations, time ordered product. Quantum statistical mechanics: density matrix, Euclidian time.

2 Introduction to Path integrals

Path integral in real and imaginary time for a free particle. Particle in a potential. Example of the harmonic oscillator. Observables and correlation functions. Euclidian quantum mechanics and statistical physics.
Nature of "quantum trajectories" and stochastic processes. The free field.

3 Critical points in statistical mechanics

Critical phenomena. Universality, scaling laws. Order parameter, symmetry breaking, critical exponents. Transfer matrices.

4 Mean field theory

Mean field theory for Ising. Landau theory. Fluctuations. Lower critical dimension. Ginzburg criterion and upper critical dimension.

5 Functional integral: the free field

Functional integral. Propagators. N-point functions and Wick's theorem. Generating functionals.

6 Φ^4 theory: perturbation theory

Functional integral for Φ^4. Perturbative expansion, Feynman graphs. Correlation functions. Effective action. Feynman amplitudes. Divergences.

7 Φ^4: one loop renormalization

UV divergences and regularization. Renormalization. $D = 4$, $D < 4$. $D = 4 - \epsilon$.

8 The Wilson RG

General ideas, simple examples. Critical points and manifolds. Critical exponents, scaling laws. Continuum limit and quantum field theory. critical phenomena and renormalization group.

9 RG: more examples and applications

$O(N)$ models, multicritical points, sigma models, geometrical problems: membranes, polymers...

SOME REFERENCES

- J. Zinn-Justin, *Quantum field theory and critical phenomena*, Oxford Science Publications. A standard reference - very dense. ISBN-13 : 978-0198509233
- G. Parisi, *Statistical Field Theory* Perseus, Reading, MA, 1998. Original and inspiring. ISBN-13 : 978-0738200514
- J. Cardy, *Scaling and renormalization in statistical physics*, Cambridge Lecture Notes in Physics 96. Very elegant. ISBN : 9780521499590 DOI :10.2277/0521499593
- E. Brzin, *Introduction To Statistical Field Theory*, Cambridge University Press. A good recent reference.ISBN-13 : 9780521193030
- M. Kardar, *Statistical Physics of Fields*, Cambridge University Press,
- R. P. Feynman and Hibbs, *Path Integrals and Quantum Mechanics*. ISBN-13 : 978-0486477220
- K. G. Wilson and J. Kogut, *The renormalization group and the epsilon-expansion*, Phys. Rep. 12C, 77 (1974)
- S. Weinberg, *The Quantum Theory of Fields*, Cambridge University Press. A remarkable book, mostly oriented towards high-energy physics.(2001)
- S. Coleman, *Aspects of symmetry*. Beautiful, inspiring and historical.