EE 592: Computational Methods for Inverse Problems
Spring 2021

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University of Southern California

Lectures: 9:00-10:50am Tue/Thu, through Zoom (find link through course website)

First Class: Tuesday, January 19th
Midterm: Thursday, March 4th
Last Class: Thursday, April 29th
Final Exam: Tuesday, May 11th, 8am-10am

Instructor: Professor Justin P. Haldar
Email: jhaldar@usc.edu
Office Hours: 2:00-3:30pm Mon/Tue, through Zoom (find link through course website)

Catalog Description: A rigorous description of vector space and functional analysis concepts and tools that are useful for solving inverse problems in real-world applications.

Prerequisites: EE 483 and EE 510

Course Overview: In many practical applications, we are interested in estimating a signal of interest based on some kind of measured data. For example, we may be interested in estimating a 3D medical image from a set of 2D X-ray projections, or estimating a high-quality image from data acquired with a noisy/blurry/low-resolution camera, or estimating the speech produced by one specific person from an audio recording of a noisy party where many people are talking simultaneously, or estimating the location of an earthquake based on information measured by an array of seismometers. These are all examples of "inverse problems," and these types of problems can be found everywhere in the modern world.

This course provides a rigorous description of vector space and functional analysis concepts and tools that are commonly used to solve modern inverse problems in a variety of real-world applications. Topics will include linear inverse problems in finite and infinite dimensional vector spaces; the existence, uniqueness, and stability of solutions to inverse problems; ill-posedness, conditioning, and regularization; Banach and Hilbert spaces; optimal design of experiments; iterative optimization methods for solving large-scale and/or nonlinear inverse problems; sparse and low-rank signal modeling; and harmonic retrieval. While the course material focuses on inverse problem contexts, the concepts, tools, and methods we discuss are also useful for solving signal approximation, signal representation, and signal design problems, and are broadly relevant to signal processing, machine learning, and optimization.

Coursework will include proving theorems, deriving methods and algorithms for solving inverse problems, and the practical application of these methods to real-world problems. Real-world application examples will be far-ranging, including artifact and noise removal in audio, image, and video signals; computational imaging (i.e., forming an image from low-quality measurements or indirect measurements); the design of optimal sensing systems; and direction-of-arrival estimation from sensor array data. Students will also undertake a substantial project on a topic they choose.

Required Texts: None.
Recommended Texts:

- D. Luenberger, *Optimization by Vector Space Methods*, Wiley, 1969.
- T. Moon and W. Sterling, *Mathematical Methods and Algorithms for Signal Processing*, Prentice Hall, 2000.
- J. A. Fessler. *Image reconstruction: Algorithms and analysis.* Unpublished book manuscript.
- C. A. Bouman. *A Guide to the Tools of Model Based Image Processing.* Unpublished book manuscript.

Grading and Course Policies:

40% Homeworks
20% Project
20% Midterm Exam
20% Final Exam

Homeworks must be turned in at the beginning of class (9:00am) on the due date. Late homeworks will receive a score of zero. The final homework grade will be based on your average score after discarding the lowest.

You are allowed (and encouraged!) to discuss homework assignments with your classmates, but are expected to complete your homework assignments individually. USC's recommended sanction for plagiarism, unauthorized collaboration, and/or cheating on any coursework is an F for the course, with a possibility for further disciplinary action.

Several of the homeworks will require MATLAB or Python programming. It is your responsibility to make sure that you know how to access the software and read/write/debug code.

Websites:
All course materials will be distributed through the USC Blackboard website: https://blackboard.usc.edu/. It is your responsibility to check the website regularly for updates (notes, assignments, due dates, etc.).

We will be using Piazza for class discussion. The system is highly catered to getting you help fast and efficiently from your classmates and myself. Rather than emailing questions to me, I encourage you to post your questions on Piazza so that everyone in the course can benefit from the discussion. The Piazza page for the course can be found at: https://piazza.com/usc/spring2021/ee592/home. If you have any problems or feedback for the developers, email team@piazza.com.

Suggestions:
My goal is to teach you and your classmates as much as possible about solving inverse problems, while simultaneously inspiring your interest, excitement, and curiosity about the material. This will be easier if you:

- Come to class on time and pay attention.
- Ask questions and participate in classroom discussion.
- Do all of the assignments.
- Make use of office hours.
- If you're struggling with the material, don't wait until the last minute to talk to me.
- Don't violate USC's academic integrity standards – you won't enjoy the consequences.

COURSE OUTLINE

Week 1: Inverse problems; analytic versus model-based solution approaches; least squares, maximum likelihood, penalized maximum likelihood, maximum *a posteriori*, minimum mean-squared error, and minimum absolute error estimation; linear vector spaces and subspaces; linear varieties, linear combinations, and linear independence.
Week 2: Bases; finite and infinite dimensional spaces; norms; existence and uniqueness of solutions in $\mathbb{R}^N$; left and right inverses; orthogonality; projectors; fundamental theorem of linear algebra.
Week 3: Least-squares solutions; minimum norm solutions; minimum norm least-squares solutions.
Week 4: Moore-Penrose pseudoinverse; singular value decomposition; matrix norms; Eckart-Young theorem and applications.
Week 5: Sensitivity and conditioning of $\mathbf{Ax} = \mathbf{b}$ with errors in both $\mathbf{A}$ and $\mathbf{b}$; sensitivity and conditioning of minimum norm least-squares and least-squares problems; SVD filtering; Tikhonov regularization.
Week 6: Total least squares and applications; Landweber iteration; conjugate gradient method.
Week 7: Gauss-Markov theorem; A-, D-, and E-optimal experiment design. MIDTERM.
Week 8: Nonlinear regularization; M-estimators and influence functions; majorize-minimize methods, expectation maximization, and iterated conditional modes.

Week 9: Sparsity-constrained inverse problems in $\mathbb{R}^N$; proofs of perfect reconstruction for ℓ_0 and ℓ_1 minimization under restricted isometry conditions; low-rank matrix completion.
Week 10: Constrained optimization; KKT conditions; penalty method; augmented Lagrangian method; ADMM; plug-and-play priors.
Week 11: Hamel bases, normed vector spaces, the $\ell_p(\mathbb{Z}^N)$ and $\mathcal{L}_p(\mathbb{R}^N)$ vector spaces; equivalence classes; inner product spaces; induced norms; parallelogram law and polarization identity.
Week 12: Linear operators; norms on linear operators; adjoints; equivalence of norms in finite dimensional spaces; matrix representations of inverse problems in finite dimensional spaces.
Week 13: Convergence of vector sequences; vector Cauchy sequences; Banach spaces; Hilbert spaces; minimum norm least-squares problems in Hilbert spaces.
Week 14: Harmonic retrieval and applications.
Week 15: Final project presentations.
Week 16: FINAL EXAMINATION

Statement on Academic Conduct and Support Systems

Academic Conduct

Plagiarism - presenting someone else's ideas as your own, either verbatim or recast in your own words - is a serious academic offense with serious consequences. Please familiarize yourself with the discussion of plagiarism in *SCampus* in Section 11, *Behavior Violating University Standards* (https://scampus.usc.edu/1100-behavior-violating-university-standards-and-appropriate-sanctions/). Other forms of academic dishonesty are equally unacceptable. See additional information in SCampus and university policies on scientific misconduct (http://policy.usc.edu/scientific-misconduct/).

Discrimination, sexual assault, and harassment are not tolerated by the university. You are encouraged to report any incidents to the *Office of Equity and Diversity* (http://equity.usc.edu/) or to the *Department of Public Safety* (http://capsnet.usc.edu/department/department-public-safety/online-forms/contact-us). This is important for the safety whole USC community. Another member of the university community - such as a friend, classmate, advisor, or faculty member - can help initiate the report, or can initiate the report on behalf of another person. *The Center for Women and Men* (http://www.usc.edu/student-affairs/cwm/) provides 24/7 confidential support, and the sexual assault resource center webpage (sarc@usc.edu) describes reporting options and other resources.

Support Systems

A number of USC's schools provide support for students who need help with scholarly writing. Check with your advisor or program staff to find out more.

Students whose primary language is not English should check with the *American Language Institute* (http://dornsife.usc.edu/ali), which sponsors courses and workshops specifically for international graduate students. *The Office of Disability Services and Programs* (http://sait.usc.edu/academicsupport/centerprograms/dsp/home_index.html) provides certification for students with disabilities and helps arrange the relevant accommodations. If an officially declared emergency makes travel to campus infeasible, *USC Emergency Information* (http://emergency.usc.edu/) will provide safety and other updates, including ways in which instruction will be continued by means of blackboard, teleconferencing, and other technology.