## Math 507A, Graduate Probability Theory I, Fall 2020

Exterior Course Website: http://www.stevenheilman.org/~heilman/507af20.html Prerequisite: MATH 525A or MATH 570.

**Course Content:** Probability spaces; distributions and characteristic functions; laws of large numbers, central limit problems; stable and infinitely divisible laws; conditional distributions.

Lecture Meeting Time/Location: Mondays, Wednesdays, and Fridays, 1PM-150PM, on zoom [link posted on blackboard]

Instructor: Steven Heilman, stevenmheilman@gmail.com

Office Hours: Mondays, 930AM-1130AM, on zoom [link posted on blackboard]

TA: A. John Rahmani, ajrahman@usc.edu

TA Office Hours: Tuesdays 2PM-4PM and Thursdays 2PM-3PM, (online in the Math Center).

**Recommended Textbook** Durrett, Probability: Theory and Examples, 4th Edition. (A draft of the book is available online here). I think this is a good book to own if you will study probability and its related fields in the future.

**Other Textbooks (not required):** I will be drawing on various sources in the course; for example, I will be drawing on some lecture notes of Tao here. These notes complement the Durrett text well. Dembo's notes available here also complement the Durrett text well.

Feller, An Introduction to Probability Theory and its Applications, Volumes 1 and 2. This set of two books is encyclopedic and very detailed, in contrast to Durrett's intentionally terse book.

Ledoux, The Concentration of Measure Phenomenon. I will include a few results from this book near the end of the course.

First Midterm: Monday, September 21, 1PM-150PM Second Midterm: Monday, October 26, 1PM-150PM

Final Exam: Wednesday, November 18, 430PM-630PM

**Other Resources:** An introduction to mathematical arguments, Michael Hutchings, An Introduction to Proofs, How to Write Mathematical Arguments

**Zoom Classroom Conduct:** Students should attend zoom lectures in a considerate way and abide by the following rules of decorum. Failure to do so could result in a diminished participation grade. It is preferable (though not required, for equity reasons) that all students have a webcam on during the lecture.

**Zoom Security:** The zoom links posted on blackboard should not be shared with anyone. You must log into zoom with your USC email address. No one will be admitted to the lecture from the "waiting room" (if you are in the waiting room, you did not log in with your USC email address).

**Zoom Technical Support:** Technical support for undergraduate students is provided through USC's ITS. Below is the contact information.

Undergraduate Student Technology Support Portal: https://itsusc.service-now.com/its sp Phone: 213-740-5555 Email: consult@usc.edu **Lecture Recording:** Zoom lectures will be recorded and posted on the blackboard site. It is USC policy to prohibit the sharing of any recording of course lectures with others. Similarly, you should not create your own recording of the lectures.

**Time Zone Issues:** If the course lectures, office hours, or exam schedules occur outside the range of 7AM-10PM in your current time zone, please alert me to this fact as soon as possible. Late notification of such an issue (e.g. the day before an exam) may result in a denied rescheduling request.

### **Email Policy:**

- My email address for this course is stevenmheilman@gmail.com.
- It is your responsibility to make sure you are receiving emails from stevenmheilman@gmail.com, and they are not being sent to your spam folder.
- Do NOT email me with questions that can be answered from this document.

**Exam Procedures:** If enrollment is ten or lower, midterm exams will be oral exams, where I ask each person questions individually, over zoom (probably for 10-20 minutes). If enrollment goes higher than ten, the midterms will be 24-hour take-home exams, to be submitted on blackboard. In the midterm exams, you are allowed to consult your homeworks, your notes, and your textbook, but these are the only resources you are allowed to use during the exams. So, you are not allowed to use the internet, internet searches, a friend or assistant, etc. Phones must be turned off. If you have anticipate issues with a stable internet connection (for obtaining the exam), issues with obtaining a suitable exam environment, etc., please let me know as soon as possible and we can try to come up with a solution to these issues. Cheating on an exam results in a score of zero on that exam. Exams can be regraded at most 15 days after the date of the exam. This policy extends to homeworks as well. All students are expected to be familiar with the USC Student Conduct Code. (See also here.)

**Disability Services:** If you are registered with disability services, I would be happy to discuss this at the beginning of the course. Any student requesting accommodations based on a disability is required to register with Disability Services and Programs (DSP) each semester. A letter of verification for approved accommodations can be obtained from DSP. Please be sure the letter is delivered to me as early in the semester as possible. DSP is located in 301 STU and is open 8:30am-5:00pm, Monday through Friday.

https://dsp.usc.edu 213-740-0776 (phone) 213-740-6948 (TDD only) 213-740-8216 (fax) ability@usc.edu

**Student Conduct:** Discrimination, sexual assault, and harassment are not tolerated by the university. You are encouraged to report any incidents to the Office of Equity and Diversity

http://equity.usc.edu/or to the Department of Public Safety http://capsnet.usc.edu/department/department-public-safety/online-forms/contact-us. This is important for the safety whole USC community. Another member of the university community - such as a friend, classmate, advisor, or faculty member - can help initiate the report, or can initiate the report on behalf of another person. The Center for Women and Men http://www.usc.edu/student-affairs/cwm/ provides 24/7 confidential support, and the sexual assault resource center webpage sarc@usc.edu describes reporting options and other resources.

## **Homework Policy:**

- Homeworks are due at **12PM noon Fridays**.
- Homeworks are submitted in blackboard, under the "Assignments" tab. You are allowed unlimited submission "attempts" for an assignment, but only the last submission will be graded. To avoid internet issues, I recommend making your first submission of an assignment well in advance of the deadline. (Note that phone tethering can also give you an internet connection to a computer.)
- Homeworks should be submitted as single PDF documents. One way to create a PDF document from paper homework assignments is the freely available Adode Scan App.
- Late homework is not accepted.
- If you still want to turn in late homework, then the number of minutes late, divided by ten, will be deducted from the score. (The time estimate is not guaranteed to be accurate.)
- Do not submit homework via email.
- The **lowest two** homework scores will be dropped. This policy is meant to account for illnesses, emergencies, dropped internet connections, etc.
- You may not use the internet to try to find answers to homework problems.
- A random subset of the homework problems will be graded each week. However, it is strongly recommended that you try to complete the entire homework assignment.
- All homework assignments must be **written by you**, i.e. you cannot copy someone else's solution verbatim. However, collaboration on homeworks is allowed and encouraged.
- Homework solutions will be posted on Friday after the homework is turned in.

# Grading Policy:

- The final course grade is weighted as the larger of the following two schemes: Scheme 1: class participation (5%), homework (35%), the first midterm (15%), the second midterm (20%), and the final (25%). Scheme 2: class participation (5%), homework (35%), the largest midterm grade (25%), and the final (35%).
- The grade for the semester will be curved. However, I do not "curve down" since anyone who exceeds my expectations in the class by showing A-level performance on the exams and homeworks will receive an A for the class.

- If you cannot attend one of the exams, you must notify me within the first two weeks of the start of the quarter. Later requests for rescheduling will most likely be denied.
- Class participation is not the same as attendance. I will never explicitly take attendance, but I will notice if someone is frequently absent. Things that increase your class participation grade include: asking good questions, paying attention in class, showing up on time or early to class, etc. Things that decrease your class participation grade include: excessive talking or disruptions during class, frequent absences, excessive texting/smartphone usage in class, frequent tardiness, etc.
- You must take the final exam to pass the course.

	Monday	 Wednesday	 Friday
1	Aug 17: Review of	Aug 19: Review of	Review of Measure Theory
	Measure Theory	Measure Theory	
2	Aug 24: 1.1, Proba-	Aug 26: 1.1, Probabil-	Aug 28: Homework 1 due. 1.2, Distri-
	bility Spaces	ity Spaces	butions
3	Aug 31: 1.3, Ran-	Sep 2: 1.3, Random	Sep 4: Homework 2 due. 1.6, Expected
	dom Variables	Variables	Value
4	Sep 7: No class	Sep 9: 1.6, Expected	Sep 11: Sep 10: Homework 3 due. 1.7
		Value	Product Measures
5	Sep 14: 2.1, Indepen-	Sep 16: 2.2, Weak Law	Sep 18: No homework due, 2.2, Weak
	dence	of Large Numbers	Law of Large Numbers
6	Sep 21: Midterm #1	Sep 23: 2.3, Borell-	Sep 25: Homework 4 due, 2.4, Strong
		Cantelli Lemmas	Law of Large Numbers
7	Sep 28: 2.4, Strong	Sep 30: 3.2, Weak Con-	Oct 2: Homework 5 due, 3.2, Weak
	Law of Large Num-	vergence	Convergence
	bers		
8	Oct 5: 3.3, Charac-	Oct 7: 3.4, Central	Oct 9: Homework 6 due, 3.4, Central
	teristic Functions	Limit Theorem	Limit Theorem
9	Oct 12: The Lin-	Oct 14: Stein's Method	Oct 16: Homework 7 due, Stein's
	deberg Replacement		Method
	Method		
10	Oct 19: 4.1, Random	Oct 21: 4.1, Random	Oct 23: No homework due, 4.1, Stop-
	Walks	 Walks	ping Times
11	Oct 26: Midterm $\#2$	Oct 28: 4.2, Recur-	Oct 30: Homework 8 due, 5.1, Condi-
		 rence	tional Expectation
12	Nov 2: 5.1, Condi-	Nov 4: 5.1, Conditional	Nov 6: Homework 9 due, 5.2, Martin-
	tional Expectation	Expectation	gales
13	Nov 9: 5.2, Martin-	Nov 11: 5.3, Martin-	Nov 13: Homework 10 due., 5.3, Mar-
	gales	gale Examples	tingale Examples

**Tentative Schedule:** (This schedule may change slightly during the course.)

# Advice on succeeding in a math class:

• Review the relevant course material **before** you come to lecture. Consider reviewing course material a week or two before the semester starts.

- When reading mathematics, use a pencil and paper to sketch the calculations that are performed by the author.
- Come to class with questions, so you can get more out of the lecture. Also, finish your homework at least **two days** before it is due, to alleviate deadline stress.
- Write a rough draft and a separate final draft for your homework. This procedure will help you catch mistakes. Also, it might be beneficial to typeset your homework. Learning LaTeX is a good skill to have for doing mathematics. Here is a template .tex file if you want to get started typesetting.
- If you are having difficulty with the material or a particular homework problem, review Polya's Problem Solving Strategies, and come to office hours.