

MATH 435: VECTOR ANALYSIS AND INTRODUCTION TO DIFFERENTIAL GEOMETRY SPRING 2020 SYLLABUS

Disclaimer: This syllabus does not constitute a contract. The instructor reserves the right to make changes at his discretion throughout the semester.

General information:

- **Prerequisites:** MATH 226 + One of the 3 courses: MATH 225, 245, or 471.
- **Lectures:** 2:00–2:50 am MWF in GFS 108. **Attendance is mandatory.**
- **Instructor:** Dr. Guillaume Dreyer
Office: KAP 258
Email: gdreyer@usc.edu
Office hours: MWF 12:00–1:00 pm
- **Teaching Assistant:** Daniel Douglas
Office: KAP 262A
Email: dcdoug1a@usc.edu
Office hours: T 3:00–5:00 pm; F 3:30–4:30 pm, in the Math Center (KAP 263)

Grading breakdown: Homework 25%; 2 Midterm exams, 20% each; Final exam 35%.

Course topics and learning objectives: Differential geometry and topology is a topic at the intersection of multiple mathematical areas: multivariable calculus, differential equations, linear algebra, real analysis, point-set and algebraic topology, and of course geometry. It is a journey that takes you a bit everywhere. Getting familiar with the language of differential geometry requires hard work, reflection, and patience. Ultimately, this course will be a unique opportunity to develop a more mature overview of mathematics. It will illustrate how different powerful mathematical theories and methods are brought together to deliver some of the finest results in mathematics. This course will also be a great opportunity to review and consolidate all the mathematical knowledge and skills you've learned so far.

- **Surfaces:** coordinate patch, atlas, differential structure on a surface, surfaces in \mathbb{R}^3 , abstract surfaces, smooth path, tangent vector, tangent plane, orientation, examples and construction of surfaces;
- **Differential calculus on surfaces:** smooth function on a surface, smooth map between surfaces, tangent map, critical points, Lagrange multipliers, Morse lemma and classification of critical points, vector field, differential form, exterior derivative, integration, singular chain, Stokes' theorem;
- **Riemannian geometry:** 1st and 2nd fundamental forms, connection, covariant derivative, parallel transport, geodesic, Gaussian curvature, locale Gauss-Bonnet formula;
- **Differential topology:** polygonal/cell decomposition of a surface, Euler-Poincare invariant, global Gauss-Bonnet formula, index of a vector field and Hopf theorem, elements of Morse theory, additional topics if time permits.

In addition to the above topics, lectures may include as needed short reviews of key concepts of linear algebra, topology, advanced multivariable calculus, differential equations.

Lecture notes and textbooks: We will not be following a specific textbook. Here are some suggested references though.

- Andrew Presley, *Elementary Differential Geometry* (can be freely downloaded on the USC Library website)
- Barrett O’Neil, *Elementary Differential Geometry* (a classic reference, very Riemannian geometry oriented)
- James Mumkres, *Topology* (classic reference in topology, widely used, with an introduction to the fundamental group and covering spaces)
- André Gramain, Leo F. Boron, *Topology of Surfaces* (the shortest path to the classification of compact surfaces via Morse theory; a very enjoyable, concise book to read once being familiar with differential geometry and topology, not easy but very inspiring)
- John W. Milnor, *Topology from the Differentiable Viewpoint* (more Graduate level; concise, clear and elegant)

Homework: Weekly homework will be posted on BB every Wednesday. **Assignment are due the following Wednesday in class at the beginning of our lecture.** Late and electronically submitted homework will not be accepted, no exceptions. You are allowed to drop one HW score. (Keep that one-time deal for that day you find yourself sick.)

You are strongly encouraged to discuss homework problems with your peers and to work in groups. This is a most efficient and rewarding way to learn and work. However, you must write your own solutions. **Homework which is simply copied from another source (friend, another textbook, internet, etc.) will be considered as plagiarism, a serious offense to USC Student Code of Conduct.**

Exams: There will be two midterms and a final.

- **Midterm 1:** Friday, February 21st, in class.
- **Midterm 2:** Friday, March 27th, in class.
- **Final:** Monday, May 11th, 2:00–4:00 pm.

If there is a scheduling conflict for an exam, **you must let me know at least 2 weeks ahead.** A scheduling conflict must involve an activity sponsored and approved by USC (marching band, athlete event, etc.). In particular, the university club or organization in question must send an official request, with the Dean’s approval, to all faculty. Personal activities do not qualify. **Failure to attend an examination will not be excused under no circumstances.**

No calculator, no cell/smart phone or other electronic device will be allowed during examinations.

Resources: The Math Center is located in KAP 263 and is open weekdays from 8 am to 7 pm (it closes earlier at 5 pm on Fridays). For up-to-date information on the consulting hours, visit the Math Center homepage <http://dornsife.usc.edu/mathcenter>. The purpose of the Math Center is to provide an environment where students can stop by to get help on their math classes. Math TAs at USC hold their office hours there. It is probably better to attend office hours of TAs who are teaching Math 126 this term. However, you are welcome to stop by the Math Center at any time and seek for help from any of the Instructors or TAs who are present at that time.

Students with disabilities: Any student requesting academic accommodations based on a disability is required to register with Disability Services and Programs (DSP) each semester and a letter of verification detailing approved accommodations must be delivered to your Instructor as early in the semester as possible. DSP is located in STU 301 and is open 8:30–5:00 pm, Monday through Friday. The phone number for the DSP office is (213) 740–0776.