

EE 592: Computational Methods for Inverse Problems

Spring 2020

Ming Hsieh Department of Electrical and Computer Engineering
University of Southern California

Lectures: 9:30-10:50am Tue/Thu, KAP 166

First Class: Tuesday, January 14th

Midterm: Tuesday, March 10th

Last Class: Thursday, April 30th

Final Exam: Tuesday, May 12th
8:00am-10:00am

Instructor: Professor Justin P. Haldar

Email: jhaldar@usc.edu

Office: EEB 442

Office Hours: 2:00-3:30pm Mon/Tue

Telephone: (213) 740-2358

Catalog Description: Vector space methods for solving inverse problems. Existence and uniqueness of solutions; conditioning and regularization; iterative algorithms; constrained optimization; applications in signal and image processing.

Prerequisites: EE 483 and EE 510

Course Overview: In many practical applications, we are interested in estimating a signal of interest based on some kind of incomplete, degraded, or indirectly measured data. For example, we may be interested in estimating a 3D medical image from a set of 2D X-ray projections, or estimating a high-quality image from data acquired with a noisy/blurry/occluded/low-resolution camera, or estimating the speech produced by one specific person from an audio recording of a noisy party where many people are talking simultaneously, or estimating the location of an earthquake based on information measured by an array of seismometers. These are all examples of “inverse problems,” and these types of problems can be found everywhere in the modern world

This course provides a rigorous description of vector space and functional analysis concepts and tools that are commonly used to solve modern inverse problems in a variety of real-world applications (e.g., artifact and noise removal in audio, image, and video signals; deblurring, tomography, and Fourier imaging; computational photography; machine learning; etc.). Topics include linear inverse problems in finite and infinite dimensional vector spaces, the singular value decomposition and the Moore-Penrose pseudoinverse, conditioning and regularization, Banach and Hilbert spaces, optimal design of experiments, iterative optimization methods for

solving large-scale and/or nonlinear inverse problems, and compressed sensing. While the course focuses on inverse problems, the concepts, tools, and methods we discuss are also useful for solving signal approximation and signal design problems.

Coursework will include proving theorems, deriving methods and algorithms for solving signal processing problems in vector spaces, and using Matlab or Python to apply these methods to real-world signal processing and data science problems. Students will also complete a final course project on a topic of their choice related to the course material.

Required Texts: None.

Recommended Texts:

- D. Luenberger, *Optimization by Vector Space Methods*, Wiley, 1969.
- T. Moon and W. Sterling, *Mathematical Methods and Algorithms for Signal Processing*, Prentice Hall, 2000.
- J. A. Fessler. *Image reconstruction: Algorithms and analysis*. Unpublished book manuscript.
- C. A. Bouman. *A Guide to the Tools of Model Based Image Processing*. Unpublished book manuscript.

Grading and Course Policies:

40% Homeworks
20% Project
20% Midterm Exam
20% Final Exam

Homeworks must be turned in at the beginning of class (9:30am) on the due date. Late homeworks will receive a score of zero. The final homework grade will be based on your average score after discarding the lowest.

You are allowed (and encouraged!) to discuss homework assignments with your classmates, but are expected to complete your homework assignments individually. USC's recommended sanction for plagiarism, unauthorized collaboration, and/or cheating on any coursework is an F for the course, with a possibility for further disciplinary action.

Several of the homeworks will require MATLAB or Python programming. It is your responsibility to make sure that you know how to access the software and read/write/debug code.

All exams are cumulative and closed book, with no calculators. Please check now for any conflicts with the scheduled exam times.

Websites:

All course materials will be distributed through the USC Blackboard website: <https://blackboard.usc.edu/>. It is your responsibility to check the website regularly for updates (notes, assignments, due dates, etc.).

We will be using Piazza for class discussion. The system is highly catered to getting you help fast and efficiently from your classmates and myself. Rather than emailing questions to me, I encourage you to post your questions on Piazza so that everyone in the course can benefit from the discussion. The Piazza page for the course can be found at: <https://piazza.com/usc/spring2020/ee592/home>. If you have any problems or feedback for the developers, email team@piazza.com.

Suggestions:

My goal is to teach you and your classmates as much as possible about solving inverse problems, while simultaneously inspiring your interest, excitement, and curiosity about the material. This will be easier if you:

- Come to class on time and pay attention.
- Ask questions and participate in classroom discussion.
- Do all of the assignments.
- Make use of office hours.
- If you're struggling with the material, don't wait until the last minute to talk to me.
- Don't violate USC's academic integrity standards – you won't enjoy the consequences.

COURSE OUTLINE

Week 1: Inverse problems; analytic versus model-based solution approaches; least squares, maximum likelihood, penalized maximum likelihood, maximum *a posteriori*, minimum mean-squared error, and minimum absolute error estimation; linear vector spaces and subspaces.

Week 2: Linear varieties, combinations, and independence; basis; finite and infinite dimensional spaces; norms; existence and uniqueness of solutions in \mathbb{R}^N ; left and right inverses.

Week 3: Orthogonality; projectors; fundamental theorem of linear algebra; least-squares solutions.

Week 4: Minimum norm solutions; minimum norm least-squares solutions; Moore-Penrose pseudoinverse.

Week 5: Singular value decomposition.

Week 6: Matrix norms; Eckart-Young theorem and applications; sensitivity and conditioning of $\mathbf{Ax} = \mathbf{b}$ with errors in both \mathbf{A} and \mathbf{b} .

Week 7: Sensitivity and conditioning of minimum norm least squares and least squares problems; SVD filtering; Tikhonov regularization; total least squares and applications.

Week 8: Landweber iteration; conjugate gradient method; Gauss-Markov theorem; A-, D-, and E-optimal experiment design.

Week 9: MIDTERM; Nonlinear regularization; M-estimators and influence functions; majorize-

minimize methods, expectation maximization, and iterated conditional modes.

Week 10: Sparsity-constrained inverse problems in \mathbb{R}^N ; proofs of perfect reconstruction for ℓ_0 and ℓ_1 minimization under restricted isometry conditions.

Week 11: Constrained optimization; KKT conditions; penalty method; augmented Lagrangian method.

Week 12: Hamel bases, normed vector spaces, the $\ell_p(\mathbb{Z}^N)$ and $\mathcal{L}_p(\mathbb{R}^N)$ vector spaces.

Week 13: Equivalence classes; inner product spaces; induced norms; parallelogram law and polarization identity; linear operators; norms on linear operators; adjoints; equivalence of norms in finite dimensional spaces.

Week 14: Matrix representations of inverse problems in finite dimensional spaces; convergence of vector sequences; vector Cauchy sequences; Banach spaces; Hilbert spaces; minimum norm least squares problems in Hilbert spaces.

Week 15: VARPRO; Stochastic methods for solving energy minimization problems (Gibbs sampling, Metropolis-Hastings algorithm).

Week 16: FINAL EXAMINATION

Statement on Academic Conduct and Support Systems

Academic Conduct

Plagiarism - presenting someone else's ideas as your own, either verbatim or recast in your own words - is a serious academic offense with serious consequences. Please familiarize yourself with the discussion of plagiarism in *SCampus* in Section 11, *Behavior Violating University Standards* (<https://scampus.usc.edu/1100-behavior-violating-university-standards-and-appropriate-sanctions/>). Other forms of academic dishonesty are equally unacceptable. See additional information in *SCampus* and university policies on scientific misconduct (<http://policy.usc.edu/scientific-misconduct/>).

Discrimination, sexual assault, and harassment are not tolerated by the university. You are encouraged to report any incidents to the *Office of Equity and Diversity* (<http://equity.usc.edu/>) or to the *Department of Public Safety* (<http://capsnet.usc.edu/department/department-public-safety/online-forms/contact-us>). This is important for the safety whole USC community. Another member of the university community - such as a friend, classmate, advisor, or faculty member - can help initiate the report, or can initiate the report on behalf of another person. *The Center for Women and Men* (<http://www.usc.edu/student-affairs/cwm/>) provides 24/7 confidential support, and the sexual assault resource center webpage (sarc@usc.edu) describes reporting options and other resources.

Support Systems

A number of USC's schools provide support for students who need help with scholarly writing. Check with your advisor or program staff to find out more.

Students whose primary language is not English should check with the *American Language Institute* (<http://dornsife.usc.edu/ali>), which sponsors courses and workshops specifically

for international graduate students. *The Office of Disability Services and Programs* (<http://sait.usc.edu/academicssupport/centerprograms/dsp/home\index.html>) provides certification for students with disabilities and helps arrange the relevant accommodations. If an officially declared emergency makes travel to campus infeasible, *USC Emergency Information* (<http://emergency.usc.edu/>) will provide safety and other updates, including ways in which instruction will be continued by means of blackboard, teleconferencing, and other technology.