EE 510: Linear Algebra for Engineering (Spring 2019)

USC Viterbi School of Engineering

Units: 4

Instructor: Mohammad Reza Rajati, PhD
PHE 414
rajati@usc.edu – Include EE 510 in subject

Office Hours: Wednesday 1:30 pm–3:00 pm

TA(s): Pratyusha Das
daspraty@usc.edu – Include EE 510 in subject
Office Hours: Tuesday 8:45 am-10:45 am
Office Location: PHE 320

Grader(s): Shiyu Mou
shiyumou@usc.edu – Include EE 510 in subject

Lecture(s): Monday, Wednesday, 4 – 5:50 pm in LVL 16
Discussion(s): Wednesday, 6 – 6:50 pm in LVL 16

Webpages: Piazza Class Page for everything except grades
and Blackboard for grades and homework submission
– All HWs, handouts, solutions will be posted in PDF format
– *Student has the responsibility to stay current with webpage material*

Prerequisites: Prior courses in multivariate calculus, linear algebra, and linear system theory.

Other Requirements: Basic computer skills (e.g., plotting, Matlab, Excel, Python, etc.).

Tentative Grading: Assignments 20%
Two Midterm Exams 40%
Final Exam 40%
Participation on Piazza* 5%

Letter Grade Distribution:

$\geq$ 93.00	A	73.00 - 76.99	C
90.00 - 92.99	A-	70.00 - 72.99	C-
87.00 - 89.99	B+	67.00 - 69.99	D+
83.00 - 86.99	B	63.00 - 66.99	D
80.00 - 82.99	B-	60.00 - 62.99	D-
77.00 - 79.99	C+	$\leq$ 59.99	F

Disclaimer: Although the instructor does not expect this syllabus to drastically change, he reserves every right to change this syllabus any time in the semester.

Note on e-mail vs. Piazza: If you have a question about the material or logistics of the class and wish to ask it electronically, please post it on the piazza page (not e-mail). You may post it anonymously if you wish. Often times, if one student has a question/comment, other also have a similar question/comment. Use e-mail with the professor, TA, graders only for issues that are specific to your individually (e.g., a scheduling issue or grade issue).

Catalogue Description: Introduction to linear algebra and matrix theory and their underlying concepts; applications to engineering problems; mathematically rigorous and foundational to other classes in communication, control, and signal processing.

Course Objectives: After successful completion of this course, the student is expected to be able to:

- describe the properties of vector spaces, linear independence, spanning sets, bases, and dimension.
- describe the properties and importance of linear transformations
- describe linear transformations using matrices.
- relate the solutions of linear equations to fundamental subspaces of linear transformations.
- describe the properties and importance of pseudo-inverse and singular value decomposition.
- describe the properties and importance of normal and inner-product spaces.
- formulate linear least squares problems using matrices and solve them.
- formulate a wide range of problems in terms of eigenvalues and eigenvectors.
- calculate canonical forms of matrices.
- describe the concepts of similarity and congruence for matrices.
- describe functions of matrices, especially the matrix exponential.
- formulate optimization problems using linear algebra.
- formulate linear difference and differential equations using linear algebra and describe the concepts of controllability and observability.
- formulate vectorization problems using Kronecker products.
- formulate engineering problems using linear algebra.

Exam Dates:

- **Midterm Exam 1:** Friday, Feb 22, 8-10 am
- **Midterm Exam 2:** Friday, March 29, 8-10 am
- **Final Exam:** Wednesday, May 1, 4:30 - 6:30 PM as set by the university

Textbooks:

- **Required Textbooks:**
  - Laub, Alan J., Matrix analysis for scientists and engineers. SIAM, 2005.
  - Meyer, Carl D., Matrix analysis and applied linear algebra. SIAM, 2000.

  The organization and approach of the course resembles to those of the first textbook. The second textbook is used as a companion for detailed discussion of some concepts and as a source for sample problems for the students.

- **Recommended Textbooks:**
  1. Ortega, James M. Matrix theory: A second course, Springer Science & Business Media, 2013.
  2. Lang, Serge. Introduction to linear algebra. Springer Science & Business Media, 2012.
  3. Rosen, Keneth H., and Malo Hautus. Handbook of linear algebra, CRC Press, 2007.
  4. Strang, Gilbert, Linear algebra and its applications, 4th ed., Thomson Learning Co., 2006.
  5. Laub, Alan J. Computational matrix analysis, SIAM, 2012.
  6. Lay, David C., and Steven R. Lay, Linear algebra and its applications, 5th ed., Addison-Wesely, 2015.

Grading Policies:

- The letter grade distribution table guarantees the *minimum* grade each student will receive based on their final score. When appropriate, relative performance measures will be used to assign the final grade, at the discretion of the instructor.
  - Final grades are non-negotiable and are assigned at the discretion of the instructor. If you cannot accept this condition, you should not enroll in this course.
  - Three of your lowest homework grades will be dropped from the final grade.
  - Half of your lower midterm grade will be dropped. For example, if you receive 80 and 32 in two midterms, your total midterm grade will be $\frac{80+0.5\times 32}{1.5} = 64$ instead of $\frac{80+32}{2} = 56$.
  - *Participation on Piazza has up to 5% extra credit, which is granted on a competetive basis *at the discretion of the instructor*.

- **Homework Policy**
  - Homework is assigned on a weekly basis. *Absolutely no late homework will be accepted. A late assignment results in a zero grade.*
  - Homework solutions should be typed or *scanned* using scanners or mobile scanner applications like CamScanner and uploaded on the course website (photos taken by cell-phone cameras and in formats other than pdf will NOT be accepted). Programs and simulation results have to be uploaded on the course website as well.

  - Students are encouraged to discuss homework problems with one another, but each student must do their own work and submit individual solutions written/ coded in their own hand. Copying the solutions or submitting identical homework sets is written evidence of cheating. The penalty ranges from F on the homework or exam, to an F in the course, to recommended expulsion.
  - Posting the homework assignments and their solutions to online forums or sharing them with other students is strictly prohibited and infringes the copyright of the instructor. Instances will be reported to USC officials as academic dishonesty for disciplinary action.

- **Exam Policy**
  - **Make-up Exams:** No make-up exams will be given. If you cannot make the above dates due to a class schedule conflict or personal matter, you must drop the class. In the case of a required business trip or a medical emergency, a signed letter from your manager or physician has to be submitted. This letter must include the contact of your physician or manager.
  - Midterms and final exams will be closed book and notes. No calculators are allowed nor are computers and cell-phones or any devices that have internet capability. One letter size cheat sheet (back and front) is allowed for the midterms. Two letter size cheat sheets (back and front) are allowed for the final.
  - All exams are cumulative, with an emphasis on material presented since the last exam.

- **Attendance:**
  - Students are required to attend all the lectures and discussion sessions and actively participate in class discussions. Use of cellphones and laptops is prohibited in the classroom. If you need your electronic devices to take notes, you should discuss with the instructor at the beginning of the semester.

Important Notes:

- Textbooks are secondary to the lecture notes and homework assignments.
- Handouts and course material will be distributed.
- Please use your USC email to register on Piazza and to contact the instructor and TAs.

Tentative Course Outline

Monday	Wednesday
Jan 7th **1** **Introduction** **Systems of Linear Equations** • Gauss and Gauss-Jordan Methods	9th **2** **Systems of Linear Equations** • Echelon Forms **Review of Elementary Linear Algebra** • Matrices • Operations on Matrices • Inverse of Matrices
14th **3** **Review of Elementary Linear Algebra** • Special Matrices • Inner Product and Orthogonality in Euclidean Space • Determinants • Properties of Determinants • Cramer's Rule	16th **4** **Algebraic Structures** • Groups and Abelian Groups • Rings and Ideals* • Fields **Vector Spaces** • Axioms • Subspaces • Elementary Properties of Vector Spaces
21st Martin Luther King Day	23rd **5** **Vector Spaces** • Spans • Linear Independence
28th **6** **Vector Spaces** • Bases • The Concept of Dimension	30th **7** **Vector Spaces** • Minkowski Sums • Direct Sums • Orthogonal Complements

Monday	Wednesday
Feb 4th **8** **Normed and Inner Product Spaces** • Norms • Normed Vector Spaces • Inner Products • Inner Product Spaces	6th **9** **Normed and Inner Product Spaces** • Norms Induced by Inner Products • Matrix Norms • Hölder Inequality • The Cauchy-Bunyakovsky-Schwartz (CBS) Inequality • CBS for The Space of Random Variables with Finite Variance* • Orthogonality and Orthogonal Complements in Inner Product Spaces • The De-Morgan Alegbra Created by Direct Sum, Intersection, and Orthogonal Complement $(\mathscr{S}(\mathcal{V}), \oplus, \cap, \varnothing, \mathcal{V}, \perp)$
11th **10** **Normed and Inner Product Spaces** • Orthogonal Sets and Bases • Gram-Schmidt Orthogonalization • The Grammian Matrix and Matrix Representation of Inner Products	13th **11** **Normed and Inner Product Spaces** • Positive Definite Matrices • Metrics and Metric Spaces • Convergence in Metric Spaces • Hilbert and Banach Spaces • Completeness

Monday	Wednesday
18th **12** **Linear Transformations** • Cartesian Products, Relations, Functions • Image and Pre-Image of A Mapping • Linear Transformations • Range and Kernel of Linear Transformations • Matrix Representation of Linear Transformations • Composition of Linear Transformations	20th **13** **Linear Transformations** • Properties of Range and Kernel • Linear Transformations in Euclidean Space and Orthogonality • Properties of Range and Kernel in Euclidean Space • Left and Right Nullspaces • The Four Fundamental Subspaces of A Linear Transformation in Euclidean Space • The Decomposition Theorem for Four Fundamental Subspaces of A Linear Transformation
25th **14** **Linear Transformations** • Properties of Range and Kernel in Euclidean Space • One-to-One and Onto Transformations • Isomorphisms • Rank and Nullity of a Linear Transformation • The Fundamental Theorem of Linear Algebra in Euclidean Spaces • Invertible Transformations • Linear Equations Revisited in The Framework of Linear Transformations	27th **15** **Linear Transformations and Orthogonality** • Projections • Orthogonal Projections • Orthogonal Projections on Four Fundamental Subspaces of A Linear Transformation • Adjoint of An Operator • Self-Adjoint Operators • Generalization of The Concept of Transpose and Symmetry • Orthogonal and Unitary Operators • Rotations, Reflections, and Scaling • The Householder Transformation and Outer Products

Monday	Wednesday
Mar 4th **16** **The Moore-Penrose Pseudo-Inverse** • Definition Using Fundamental Subspaces of Linear Transformations • Penrose's Algebraic Characterization of Pseudo-Inverse • Properties of Pseudo-Inverse • Orthogonal Projections on Four Fundamental Subspaces of Linear Transformations **Linear Least Squares Problems** • Problem Formulation • Relationship with Pseudo-Inverse* • Linear Regression	6th **17** **Eigenvalues and Eigenvectors** • Left and Right Eigenvectors of A Matrix • Characteristic Polynomials and Spectra • The Cayley-Hamilton Theorem • Algebraic and Geometric Multiplicities • Defective Matrices • Minimal Polynomials • Eigenvalues of Symmetric and Hermitian Matrices
11th Spring Recess	13th Spring Recess
18th **18** **Eigenvalues and Eigenvectors** • Diagonalization • Equivalent, Similar, and Congruent Matrices • The Concept of Invariance • Complex and Real Jordan Canonical Forms	20th **19** **Eigenvalues and Eigenvectors** • Principal Vectors and Jordan Forms • Jordan Blocks and Minimal Polynomials • Analytic Functions of Matrices • Techniques for Computing Functions of Matrices • Other Canonical Forms*
25th **20** **Eigenvalues and Eigenvectors** • Analytic Functions of Matrices • Techniques for Computing Functions of Matrices • Other Canonical Forms*	27th **21** **Eigenvalues and Eigenvectors** • Spectral Representation of Matrices • Congruence and Sylvester Law of Inertia • Definite Matrices

<table>
<tr><th>Monday</th><th>Wednesday</th></tr>
<tr><td>Apr 1st 22
Singular Value Decomposition
The Fundamental Theorem
Properties
SVD and Fundamental Subspaces of a Linear Transformation
SVD and The Fundamental Theorem of Linear Algebra
Application: SVD for Classification*
Principal Component Analysis*</td><td>3rd 23
Numerical Linear Algebra*
Matrix Norms and The Condition Number of A Matrix
Gram-Schmidt Orthogonalization and QR Factorization
Iterative/Numerical Solutions to Linear Equations and Eigenvalue Problems</td></tr>
<tr><td>8th 24
Review of Multivariable Calculus
Taylor Expansion of Multivariable Functions, Gradient, and Hessian
Directional Derivatives
Strong and Weak Extrema and Saddle Points of Multivariable Functions
Bilinear and Quadratic Forms
The Relationship between The Eigenstructure of Hessian Matrices and Types of Extrema
Extrema of Quadratic Forms
Rayleigh Quotient</td><td>10th 25
Application: Optimization
Steepest Descent
Newton’s Method
Conjugate Gradient*
Quasi-Newton Methods*
The Broyden-Fletcher-Goldfarb-Shanno (BFGS) Algorithm*
The Levenberg-Marquardt Algorithm*
The Nelder-Mead Algorithm*
Constrained Optimization
Karush-Kuhn-Tucker (KKT) Conditions</td></tr>
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MONDAY	WEDNESDAY
15th **26** **Application: Linear Programming and Game Theory*** • Linear Inequalities • Feasible Set and The Cost Function • Slack Variables • The Simplex Method • Dual and Primal Problems • Applications to Network Problems • Game Theory and The Minimax Theorem	17th **27** **Application: Analysis of Linear Dynamical Systems** • Linear Differential Equations • Systems of Homogeneous Linear Differential Equations • State Transition Matrix and The Convolution Integral • The Laplace Transform • Properties of The Matrix Exponential • Non-Homogeneous Equations • Modal Decomposition • Transfer Functions for Continuous-Time Linear Systems • Canonical Controllable and Observable Realizations of Linear Systems*

<table>
<tr><th>Monday</th><th>Wednesday</th></tr>
<tr><td>22nd 28

Application: Stability Analysis of Dynamical Systems

• Asymptotic Stability
• Stability in the Sense of Lyapunov
• Stability of Linear Time-Invariant Systems
• Lyapunov Equation
• Linear Difference Equations*
• Discrete-Time Linear Systems*
• Modal Decomposition of Discrete-Time Systems*
• Asymptotic Stability of Discrete-Time Systems*
• Stability of Discrete-Time Systems in The Sense of Lyapunov*
• Linear Difference Equations*
• Discrete-Time Linear Systems*
• Modal Decomposition of Discrete-Time Systems*
• Transfer Functions for Discrete-Time Linear Systems*</td><td>24th 29

Tensor Analysis

• Kronecker (Tensor) Products
• Properties of Tensor Products
• Multilinear Algebra*
• Exterior Products and Cross Products*
• Exterior Algebras and Determinants*
• Application of Kronecker Products in Solving Sylvester and Lyapunov Equations
• Tensor SVD*

Application: Tensor Analysis in Machine Learning*

• Tensor Analysis for Image Processing*
• Tensor Analysis for Deep Learning*</td></tr>
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Notes:
- Items marked by * will be covered only if time permits.

Statement on Academic Integrity: USC seeks to maintain an optimal learning environment. General principles of academic honesty include the concept of respect for the intellectual property of others, the expectation that individual work will be submitted unless otherwise allowed by an instructor, and the obligations both to protect one's own academic work from misuse by others as well as to avoid using another's work as one's own. All students are expected to understand and abide by these principles. SCampus, the Student Guidebook, contains the University Student Conduct Code (see University Governance, Section 11.00), while the recommended sanctions are located in Appendix A. See: http://scampus.usc.edu.

Emergency Preparedness/Course Continuity in a Crisis In case of a declared emergency if travel to campus is not feasible, USC executive leadership will announce an electronic way for instructors to teach students in their residence halls or homes using a combination of Blackboard, teleconferencing, and other technologies. See the university's site on Campus Safety and Emergency Preparedness: http://preparedness.usc.edu

Statement for Students with Disabilities: Any student requesting academic accommodations based on a disability is required to register with Disability Services and Programs (DSP) each semester. A letter of verification for approved accommodations can be obtained from DSP. Please be sure the letter is delivered to me (or to TA) as early in the semester as possible. DSP is located in STU 301 and is open 8:30 a.m.5:00 p.m., Monday through Friday. Website: http://sait.usc.edu/academicsupport/centerprograms/dsp/home_index.html

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