USC Viterbi School of Engineering

EE 503: Probability for Electrical and Computer Engineers (Spring 2018)

Course Description:	Rigorous coverage of probability, discrete and continuous random variables, functions of multiple random variables, covariance, correlation, random sequences, Markov chains, estimation, introduction to statistics.
Instructor:	P. Vijay Kumar
Class:	MW 2-4:50, THH 212
Office Hours:	F 1:45 - 2:45 pm and 4:15 to 5:15 pm, EEB 504B
email:	vijayk@usc.edu – (include EE 503 in the subject line of your em)
Discussion Session:	F 3-3:50 pm (THH 208)
TA 1:	Kaidong Wang, RTH 418
Office Hours:	4:15pm to 5:15 pm Friday, RTH 418
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TA 2:	Haoqi Li, EEB B16
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Grader 2:	Palash Manishkumar Shah (Palash Shah)
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Website:	USC Blackboard, Piazza
Prereqs:	Multivariable calculus (Math 445 or equivalent) Fundamentals of Linear System Theory (EE301 or equivalent)
Other Requirements:	Basic computer skills (i.e., simple programs and plotting).
Exams, HW:	15% Homework, weekly 22.5% Midterm Exam 1, Wednesday, February 14 22.5% Midterm Exam 2, Friday, March 23 (During Discussion Session) 22.5% Midterm Exam 3, Friday, April 20 (During Discussion Session) (best 2 of 3 midterm scores will be retained) 40% Final Exam, May 7, 2-4 pm (2 hours) (as per final exam schedule) - each exam is closed book, no "formula sheet"

Course Objective: To understand the basic tools of probability and random variables so as to provide a basis for analysis and design in subsequent electrical and computer engineering classes. Suitable for any engineering or applied science discipline, including computer science and financial engineering.

Grading Policies:

- Homework will be assigned every week on Wednesday, and due the following Wednesday.
- **Late HW** will not be accepted. A late assignment results in a zero grade.

- **Make-up Exams:** No make-up exams will be given. If you cannot make the above dates due to a class schedule conflict, you must notify me by the last day to add/drop. If I cannot accommodate your schedule, you must drop the class. In the case of a required business trip or medical emergency, a signed letter from your manager or doctor is required. This letter must include the telephone number of your doctor or supervisor. I must be notified as soon as possible in the case of an emergency.
- **Attendance:** Lecture attendance is encouraged but not mandatory. However, students are responsible for all material presented in lecture.

Statement on Academic Conduct: Plagiarism – presenting someone elses ideas as your own, either verbatim or recast in your own words – is a serious academic offense with serious consequences. Please familiarize yourself with the discussion of plagiarism in SCampus in Section 11, Behavior Violating University Standards https://scampus.usc.edu/1100-behavior-violating-university-standards-and-appropriate-sanctions/. Other forms of academic dishonesty are equally unacceptable. See additional information in SCampus and university policies on scientific misconduct, http://policy.usc.edu/scientific-misconduct/.

Discrimination, sexual assault, and harassment are not tolerated by the university. You are encouraged to report any incidents to the Office of Equity and Diversity http://equity.usc.edu/ or to the Department of Public Safety http://capsnet.usc.edu/department/department-public-safety/online-forms/contact-us. This is important for the safety whole USC community. Another member of the university community – such as a friend, classmate, advisor, or faculty member – can help initiate the report, or can initiate the report on behalf of another person. The Center for Women and Men http://www.usc.edu/student-affairs/cwm/ provides 24/7 confidential support, and the sexual assault resource center webpage sarc@usc.edu describes reporting options and other resources.

Statement for Students with Disabilities: A number of USCs schools provide support for students who need help with scholarly writing. Check with your advisor or program staff to find out more. Students whose primary language is not English should check with the American Language Institute http://dornsife.usc.edu/ali, which sponsors courses and workshops specifically for international graduate students. The Office of Disability Services and Programs http://sait.usc.edu/academicsupport/centerprograms/dsp/home_index.html provides certification for students with disabilities and helps arrange the relevant accommodations. If an officially declared emergency makes travel to campus infeasible, USC Emergency Information http://emergency.usc.edu/will provide safety and other updates, including ways in which instruction will be continued by means of blackboard, teleconferencing, and other technology.

Textbooks:

- **Required:**
  1. Dimitri P. Bertsekas and John N. Tsitsiklis, *Introduction to Probability*, 2nd edition, by Athena Scientific, 2008.
  2. J. A. Gubner, *Probability and Random Processes for Electrical and Computer Engineers*, Cambridge University Press, 2006.
- **Recommended:**
  1. A. Leon-Garcia, *Probability, Statistics, and Random Processes for Electrical Engineering*, 3rd Edition, Addison Wesley, 2012.

Course Outline

1. Introduction to Probability

   - example applications, how probability theory achieves its results,
   - set operations, set cardinality, functions, inverse images,
   - probability models, sample space, σ-algebra of events of interest, axioms of probability, derived properties, limit properties of the probability function,
   - conditional probability, total probability theorem, Bayes's theorem, independence of events,
   - the basic principle of counting, ordered and unordered sampling with and without replacement, partitions.

2. Discrete Random Variables

   - the probability space, random variables, discrete random variables, probability mass function (pmf), example distributions,
   - functions of a RV,
   - expectation and variance of a RV, the expected-value rule, properties of the mean and the variance,
   - joint pmf of 2 RVs, marginalization, functions of 2 RVs, expected-value rule,
   - conditioning of a RV on an event, conditional pmf, conditioning of a RV on a second RV, multiplication rule, total probability theorem,
   - conditional expectation, total expectation theorem,
   - independence of RVs, variance of the sum of RVs,
   - convergence of deterministic sequences: properties of convergence, subsequences, Cauchy convergence, lim sup, lim inf, convergence of some special sequences,
   - convergence of deterministic series: Cauchy convergence criterion, series with non-negative terms, comparison test for convergence, other tests including the root and ratio test, absolute convergence of a series, convergence under rearrangement of terms.

3. General Random Variables

   - continuous RVs: probability density function and properties,
   - expectation, mean, variance, moments, expected-value rule, example RVs,
   - CDF, Borel σ-algebra, properties of the CDF, examples,
   - normal RV, mean, variance,
   - joint CDF of 2 RVs, marginalization, expectation,
   - conditional pdf, conditioning on various events, total probability theorem,
   - conditioning one RV on a second RV,
   - conditional expectation, total expectation theorem,
   - independence of continuous RVs, independent normal RVs,
   - continuous Bayes' rule.

4. Further Topics on Random Variables

   - functions of a RV, various examples, functions of 2 RVs, sums of independent RVs,
   - covariance and correlation coefficient,
   - conditional expectation as a RV, example application, law of iterated expectation,
   - moment-generating function (MGF) as a transform, impact of linear transformation, MGF of a normal RV, recovering moments from the MGF, inversion of the transform, MGF of a sum of independent RVs, sum of independent Poisson RVs, sum of independent Gaussian RVs,
   - the characteristic function, inversion formula, moments of the normal RV, characteristic function of the normal RV, recovering moments from the characteristic function.

5. Limit Theorems

   - bounds on probabilities: Markov, Chebyshev, Chernoff,
   - Weak Law of Large Numbers (WLLN), application,
   - convergence in probability, examples,
   - the Central Limit Theorem (CLT), applications, De-Moivre Laplace approximation to the binomial,
   - Strong Law of Large Numbers (SLLN),
   - convergence with probability 1 (almost-sure (a.s.) convergence), implies convergence in probability,
   - Levi's continuity theorem, convergence in the mean square, convergence in distribution, relationship between the different modes of convergence,

6. Random Vectors

   - review of basic linear algebra: matrices, vectors, rank, inverse, solution of linear equations, eigenvalues, operation involving block matrices,
   - expectation of random vectors and matrices, the correlation matrix,
   - implication of zero variance, Cauchy-Schwarz inequality,
   - covariance matrix and properties, cross-covariance matrix, uncorrelated random vectors,
   - joint characteristic function of a RV, case when the RVs are independent,
   - KL expansion of a RV, transformation (one-one and onto) of RVs, case of a linear transformation,

7. Estimation

   - Linear Minimum Mean-Square Estimation (LMMSE) of a random vector, orthogonality principle, covariance of the error, example,
   - (Nonlinear) Minimum Mean-Square Estimation (MMSE): conditional expectation as the best estimator, orthogonality principle, error variance, example,
   - the Maximum Aposteriori Probability (MAP) estimator, the Maximum-Likelihood (ML) Estimator, example.

8. Gaussian Random Vector

- definition of a Gaussian random vector, characteristic function of a Gaussian ransom vector, linear transformation of a Gaussian random vector,
- joint pdf of a Gaussian random vector - case of nonsingular covariance matrix,
- the conditional pdf is also normal, joint pdf for the bivariate case expressed in terms of the correlation coefficient.

9. Markov Chains

- definition of a Markov Chain (MC), transition probability matrix, transition probability graph, examples,
- probability of a path, n-step transition probabilities, Chapman Kolmogorov equation, stationary distributions,
- classification of states in the state diagram, recurrent states, transient states,
- recurrence classes of states, decomposition of the MC,
- periodicity of a recurrence class, identifying aperiodic MCs, existence of steady-state probabilities,
- birth-and-death processes, balance equation, examples: random walk,
- transient behavior: example: Gambler's ruin,
- expected time to absorption, an example.

10. Statstics

- terminology, sample mean and variance, biased and unbiased estimators, consistent and strongly consistent estimators,
- parameter estimation, examples,
- estimating the distribution using a histogram, example, ch-squared, goodness-of-fit test,
- the Gamma distribution, MGF, moments, chi-square distribution as a special case, relation to Gaussian RVs, degrees of freedom, moments,
- confidence interval, confidence level, confidence intervals for the mean, known and unknown variance cases, example,
- impact of estimate when sampling with and without replacement,
- confidence intervals for Gaussian data, for the mean, known and unknown variance case,
- confidence interval for the variance known and unknown mean case,