COURSE OUTLINE

Title: PTE 599 Computational Geomechanics

Description: This graduate-level course covers physical, mathematical and simulation aspects of coupled fluid flow and geomechanics in petroleum reservoirs. Conservation laws of mass and momentum applicable to fluid-saturated porous media will be derived. Finite element and finite volume methods for solution of the coupled problem of fluid flow and deformation will be developed. Computer implementation of the numerical methods will be emphasized. Requires programming in a language of user's choice e.g. Matlab, Python, C++, Fortran.

Instructor: Birendra Jha

Meeting Dates: Tue 2-4:40 pm

Place: OHE 100C

Text: There is no prescribed textbook. Lecture notes and other materials will be provided.

References:

1. P. Chadwick, Continuum Mechanics: Concise Theory and Problems, Dover, London, 1999
2. J. Bear, Dynamics of Fluids in Porous Media, Dover, NY, 1972
3. H. F. Wang, Theory of Linear Poroelasticity with Applications to Geomechanics and Hydrogeology, Princeton University Press, NJ, 2000
4. O. C. Zienkiewicz and R. L. Taylor, The Finite Element Method vol 1: The Basis, vol 2: Solid Mechanics, vol 3: Fluid dynamics, Butterworth-Heinemann, MA, 2000
5. O. Coussy, Poromechanics, John Wiley & Sons, NJ, 2004.
6. H. F. Wang and M. P. Anderson, Introduction to Groundwater Modeling: Finite Difference and Finite Element Methods, Academic Press, 1995
7. M. D. Zoback, Reservoir Geomechanics, Cambridge University Press, 2007

Grading:

Homeworks (weekly, due at the beginning of class)	40%
Midterm (in class)	20%
Final (take home)	40%

CLASS SCHEDULE

Week 1
Course introduction. Importance of geomechanics: compaction and subsidence, hydraulic fracturing, induced earthquakes, well failure. Role of computational methods.
Introduction to continuum mechanics: kinematics of deformation, reference and spatial configurations. Tensor algebra.
Reading: Reference 3, chapter 1; Reference 1, chapter 1 and 2

Week 2
Deformation and strain, the Cauchy stress tensor and traction vector, principal and deviatoric stresses, stress invariants. Derivation of momentum conservation laws, constitutive equation for linear elasticity, displacement formulation of the equilibrium equation, compatibility relations
Reading: Reference 1, chapter 3 and 4

Week 3
Formulation of boundary value problems of 1D elasticity, the weak form, finite element approximation
Reading: Reference 4, vol 1, chapter 3

Week 4
2D elasticity. Derivation of the equilibrium, constitutive, and compatibility equations for plane stress
Reading: Reference 3, chapter 7

Week 5
2D elasticity. Derivation of the equilibrium, constitutive, and compatibility equations for plane strain
Reading: Reference 4, vol 1, chapter 4

Week 6
2D elasticity. Formulation of the matrix-vector problem for plane strain and its computer implementation
Reading: Reference 4 (vol 1 or 2), chapter 2

Week 7
Conservation laws, the effective stress, Darcy's law, drained and undrained behavior, poroelastic constants
Reading: Reference 3, chapter 3 and 4; Reference 5, chapter 4; Reference 2, chapter 6

Week 8
Biot's theory of linear poroelasticity. Coupling mechanisms between the flow and deformation problems and coupling strength. Assumption of rock compressibility in reservoir simulators.
Reading: Reference 3, chapter 6; Reference 5, chapter 5.

Detournay and Cheng, “Fundamentals of poroelasticity,” chapter 5 in Comprehensive Rock Engineering: Principles, Practice and Projects, Vol. II, Analysis and Design Method, ed. C. Fairhurst, Pergamon Press, pp. 113-171, 1993

MIDTERM EXAMINATION

Week 9
Formulation of a uniaxial consolidation problem and its analytical solution. Strong form and weak form. Finite volume approximation using the two-point flux approximation. Generalized trapezoidal rule for time integration.
Reading: Leveque’s Finite Volume Methods for Hyperbolic Problems, Cambridge University Press, chapter 1. Reference 6, chapter 2 and 4

Week 10
Finite volume approximation for solution of a single-phase flow problem in 2D. Strong and weak form. Matrix-vector formulation.
Reading: Class notes

Week 11
Implicit and explicit time integration schemes. Accuracy and stability of the forward and backward Euler integration schemes
Reading: Class notes.

Week 12
Formulation of Mandel’s 2D compaction problem. Analytical solution. The Mandel-Cryer effect and role of two-way coupling
Reading: Reference 3, chapter 7

Week 13
Mixed finite element–finite volume approximation for solution of the Mandel problem. Strong and weak forms. Matrix-vector formulation.
Reading: Reference 4 (vol 1) chapter 11 and 12

Week 14
Tectonic stresses and principal stresses. Andersonian theory and three types of faulting. Stresses induced on a fault due to production and injection of fluids.
Reading: Reference 7, chapter 1 and 2
Zoback et al, Global patterns of tectonic stress, Nature 1989
Anderson, The dynamics of faulting, Geol. Soc. London 1905.
Settari and Maurits, A coupled reservoir and geomechanical simulation system, Soc. Pet. Eng. J 1998

Week 15
Fracture mechanics. Tensile and shear hydraulic fractures. Graphical method of Mohr’s circle to determine failure along a plane
TAKE-HOME FINAL EXAMINATION

Statement on Academic Conduct and Support Systems

Academic Conduct
Plagiarism – presenting someone else's ideas as your own, either verbatim or recast in your own words – is a serious academic offense with serious consequences. Please familiarize yourself with the discussion of plagiarism in *SCampus* in Section 11, *Behavior Violating University Standards* https://scampus.usc.edu/1100-behavior-violating-university-standards-and-appropriate-sanctions. Other forms of academic dishonesty are equally unacceptable. See additional information in *SCampus* and university policies on scientific misconduct, http://policy.usc.edu/scientific-misconduct.

Discrimination, sexual assault, and harassment are not tolerated by the university. You are encouraged to report any incidents to the *Office of Equity and Diversity* http://equity.usc.edu or to the *Department of Public Safety* http://adminopsnet.usc.edu/department/department-public-safety. This is important for the safety of the whole USC community. Another member of the university community – such as a friend, classmate, advisor, or faculty member – can help initiate the report, or can initiate the report on behalf of another person. *The Center for Women and Men* http://www.usc.edu/student-affairs/cwm/ provides 24/7 confidential support, and the sexual assault resource center webpage http://sarc.usc.edu describes reporting options and other resources.

Honor Code
Engineering enables and empowers our ambitions and is integral to our identities. In the Viterbi community, accountability is reflected in all our endeavors.
Engineering + Integrity.
Engineering + Responsibility.
Engineering + Community.
Think good. Do better. Be great.
These are the pillars we stand upon as we address the challenges of society and enrich lives.

Support Systems
A number of USC's schools provide support for students who need help with scholarly writing. Check with your advisor or program staff to find out more. Students whose primary language is not English should check with the *American Language Institute* http://dornsife.usc.edu/ali, which sponsors courses and workshops specifically for international graduate students. *The Office of Disability Services and Programs* http://sait.usc.edu/academicsupport/centerprograms/dsp/home_index.html provides certification for students with disabilities and helps arrange the relevant accommodations. If an officially declared emergency makes travel to campus infeasible, *USC Emergency Information* *http://emergency.usc.edu* will provide safety and other updates, including ways in which instruction will be continued by means of blackboard, teleconferencing, and other technology.