Abstract
Quantum mechanics is the basis for understanding physical phenomena on the atomic and nanometer scale. There are numerous applications of quantum mechanics in biology, chemistry and engineering. Those with significant economic impact include semiconductor transistors, lasers, quantum optics and photonics. As technology advances, an increasing number of new electronic and opto-electronic devices will operate in ways that can only be understood using quantum mechanics. Over the next twenty years fundamentally quantum devices such as single-electron memory cells and photonic signal processing systems will become common-place. The purpose of this course is to cover a few selected applications and to provide a solid foundation in the tools and methods of quantum mechanics. The intent is that this understanding will enable insight and contributions to future, as yet unknown, applications.

Prerequisites
Mathematics:
A basic working knowledge of differential calculus, linear algebra, statistics and geometry.
Computer skills:
An ability to program numerical algorithms in MATLAB or similar language and display results in graphical form.
Physics background:
Should include a basic understanding of Newtonian mechanics, waves and Maxwell’s equations.
Introduction: Lectures 1 - 3

Lecture 1

REVIEW OF CLASSICAL CONCEPTS
The linear and nonlinear oscillator
Electromagnetism

TOWARDS QUANTUM MECHANICS – PARTICLES AND WAVES
Diffraction, interference, and correlation functions for light
Black-body radiation and evidence for quantization of light
Photoelectric effect and the photon particle

Lecture 2

Secure quantum communication
The link between quantization of photons and quantization of other particles
Diffraction and interference of electrons
When is a particle a wave?

Lecture 3

THE SCHRÖDINGER WAVE EQUATION
The wave function description of an electron of mass $m_0$ in free-space
The electron wave packet and dispersion
The Bohr model of the hydrogen atom
  Calculation of the average radius of an electron orbit in hydrogen
  Calculation of energy difference between electron orbits in hydrogen
Periodic table of elements
Crystal structure
  Three types of solid classified according to atomic arrangement
  Two-dimensional square lattice, cubic lattices in three-dimensions
Electronic properties of semiconductor crystals
  The semiconductor heterostructure

Using the Schrödinger wave equation: Lectures 4 - 6

Lecture 4

INTRODUCTION
The effect of discontinuities in the wave function and its derivative

WAVE FUNCTION NORMALIZATION AND COMPLETENESS

INVERSION SYMMETRY IN THE POTENTIAL
Particle in a one-dimensional square potential well with infinite barrier energy

NUMERICAL SOLUTION OF THE SCHRÖDINGER EQUATION
Matrix solution to the descretized Schrödinger equation
Nontransmitting boundary conditions. Periodic boundary conditions

CURRENT FLOW
Current flow in a one-dimensional infinite square potential well
Current flow due to a traveling wave

DEGENERACY IS A CONSEQUENCE OF SYMMETRY
Bound states in three-dimensions and degeneracy of eigenvalues
Lecture 5

BOUND STATES OF A SYMMETRIC SQUARE POTENTIAL WELL
Symmetric square potential well with finite barrier energy

TRANSMISSION AND REFLECTION OF UNBOUND STATES
Scattering from a potential step when effective electron mass changes
Probability current density for scattering at a step
Impedance matching for unity transmission

Lecture 6

PARTICLE TUNNELING
Electron tunneling limit to reduction in size of CMOS transistors

THE NONEQUILIBRIUM ELECTRON TRANSISTOR

Scattering in one-dimension: The propagation method: Lectures 7 - 10

Lecture 7

THE PROPAGATION MATRIX METHOD
Writing a computer program for the propagation method
TIME REVERSAL SYMMETRY
CURRENT CONSERVATION AND THE PROPAGATION MATRIX

Lecture 8

THE RECTANGULAR POTENTIAL BARRIER
Tunneling

RESONANT TUNNELING
Localization threshold
Multiple potential barriers

THE POTENTIAL BARRIER IN THE δ-FUNCTION LIMIT

Lecture 9

ENERGY BANDS IN PERIODIC POTENTIALS: THE KRONIG-PENNY POTENTIAL
Bloch’s theorem
Propagation matrix in a periodic potential

Lecture 10

THE TIGHT BINDING MODEL FOR ELECTRONIC BAND STRUCTURE
Nearest neighbor and long-range interactions
Crystal momentum and effective electron mass

USE OF THE PROPAGATION MATRIX TO SOLVE OTHER PROBLEMS IN ENGINEERING

THE WKB APPROXIMATION
Tunneling
Related mathematics: Lecture 11 - 12

Lecture 11

ONE PARTICLE WAVE FUNCTION SPACE
PROPERTIES OF LINEAR OPERATORS
Hermitian operators
Commutator algebra
DIRAC NOTATION
MEASUREMENT OF REAL NUMBERS
Time dependence of expectation values. Uncertainty in expectation value
The generalized uncertainty relation
THE NO CLONING THEOREM

Lecture 12

DENSITY OF STATES
Density of states of particle mass \( m \) in 3D, 2D, 1D and 0D
Quantum conductance
Numerically evaluating density of states from a dispersion relation
Density of photon states

The harmonic oscillator: Lectures 13 - 14

Lecture 13

THE HARMONIC OSCILLATOR POTENTIAL
CREATION AND ANNIHILATION OPERATORS
The ground state. Excited states
HARMONIC OSCILLATOR WAVE FUNCTIONS
Classical turning point
TIME DEPENDENCE
The superposition operator. Measurement of a superposition state

Lecture 14

Time dependence in the Heisenberg representation
Charged particle in harmonic potential subject to constant electric field
ELECTROMAGNETIC FIELDS
Laser light
Quantization of an electrical resonator
Quantization of lattice vibrations
Quantization of mechanical vibrations

Fermions and Bosons: Lecture 15 - 16

Lecture 15

INTRODUCTION
The symmetry of indistinguishable particles. Slater determinant
Pauli exclusion principle. Fermion creation and annihilation operators – application to tight-binding Hamiltonian
Lecture 16

FERMI-DIRAC DISTRIBUTION FUNCTION
Equilibrium statistics
Writing a computer program to calculate the Fermi-Dirac distribution

BOSE-EINSTIEN DISTRIBUTION FUNCTION
CURRENT AS FUNCTION OF VOLTAGE BIAS
Semiconductor heterostructure diode structures in the depletion approximation.
Metal-insulator-metal.
Reduced dimensions

Time dependent perturbation theory and the laser diode: Lectures 17 - 21

Lecture 17

FIRST-ORDER TIME-DEPENDENT PERTURBATION THEORY
Abrupt change in potential
Time dependent change in potential

CHARGED PARTICLE IN A HARMONIC POTENTIAL
FIRST-ORDER TIME-DEPENDENT PERTURBATION

Lecture 18

FERMI’S GOLDEN RULE
IONIZED IMPURITY ELASTIC SCATTERING RATE IN GaAs
The coulomb potential. Linear screening of the coulomb potential
Correlation effects in position of dopant atoms
Calculating the electron mean free path

Lecture 19

EMISSION OF PHOTONS DUE TO TRANSITIONS BETWEEN ELECTRONIC STATES
Density of optical modes in three dimensions
Light intensity
Background photon energy density at thermal equilibrium
Fermi’s golden rule for stimulated optical transitions
The Einstein A and B coefficients
Occupation factor for photons in thermal equilibrium in a two-level system
Derivation of the relationship between spontaneous emission rate and gain

Lecture 20

THE SEMICONDUCTOR LASER DIODE
Spontaneous and stimulated emission
Optical gain in a semiconductor. Optical gain in the presence of electron scattering

DESIGNING A LASER CAVITY
Resonant optical cavity. Mirror loss and photon lifetime
The Fabry-Perot laser diode. Rate equation models

Lecture 21
NUMERICAL METHOD OF SOLVING RATE EQUATIONS
The Runge-Kutta method. Large-signal transient response. Cavity formation

NOISE IN LASER DIODE LIGHT EMISSION
Effect of photon and electron number quantization
Langevin and semiclassical master equations

QUANTUM THEORY OF LASER OPERATION
Density matrix
Single and multiple quantum dot, saturable absorber

Time independent perturbation theory: Lectures 22 - 23

Lecture 22
NON-DEGENERATE CASE
Hamiltonian subject to perturbation \( W \)
First-order correction. Second order correction
Harmonic oscillator subject to perturbing potential in \( x, x^2 \) and \( x^3 \)

Lecture 23
DEGENERATE CASE
Secular equation
Two states
Perturbation of two-dimensional harmonic oscillator
Perturbation of two-dimensional potential with infinite barrier

Angular momentum and the hydrogenic atom: Lectures 24 - 26

Lecture 24
ANGULAR MOMENTUM
Classical angular momentum
The angular momentum operator
Eigenvalues of the angular momentum operators \( L_z \) and \( L^2 \)
Geometric representation

Lecture 25
SPHERICAL COORDINATES, SPHERICAL HARMONICS AND THE HYDROGEN ATOM
Spherical coordinates and spherical harmonics
The rigid rotator
Quantization of the hydrogenic atom
Radial and angular probability density

Lecture 26
ELECTROMAGNETIC RADIATION
No eigenstate radiation
Superposition of eigenstates
Hydrogenic selection rules for dipole radiation
Fine structure
Hybridization