EE 592: Computational Methods for Inverse Problems
Fall 2015

Ming Hsieh Department of Electrical Engineering
University of Southern California

Lectures: 9:30-10:50am Tue/Thu, KAP 147

First Class: Tuesday, August 25th

Midterm: Tuesday, October 20th

Last Class: Thursday, December 3rd

Final Exam: Thursday, December 10th
11:00am-1:00pm

Instructor: Professor Justin P. Haldar
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Office Hours: 2:00-3:30pm Mon/Thur
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Catalog Description: Vector space methods for solving inverse problems. Existence and uniqueness of solutions; conditioning and regularization; iterative algorithms; constrained optimization; applications in signal and image processing.

Prerequisites: EE 483 and EE 441
Recommended Preparation: EE 503

Course Overview: In many practical applications, the data we observe is incomplete, has been degraded during the data acquisition process (e.g., noise, out-of-focus blur, etc.), or may be indirectly related to the signals we are interested in (e.g., biomedical imaging, seismic imaging, synthetic aperture radar, and similar techniques frequently make Fourier-domain or projection-domain measurements). Often, we want to be able to estimate the original signal of interest from the measured data. This is called an "inverse problem," and inverse problems can be found everywhere in modern signal processing and data science.

This course provides a rigorous description of vector space and functional analysis concepts and tools that are commonly used to solve modern inverse problems in a variety of real-world applications (e.g., artifact and noise removal in audio, image, and video signals; deblurring, tomography, and Fourier imaging; computational photography; machine learning; etc.). Topics include linear inverse problems in finite and infinite dimensional vector spaces, the singular value decomposition and the Moore-Penrose pseudoinverse, conditioning and regularization, Banach and Hilbert spaces, optimal design of experiments, iterative optimization methods for solving large-scale and/or nonlinear inverse problems, and compressed sensing. While the course focuses on inverse problems, the concepts, tools, and methods we discuss are also useful for solving signal approximation and signal design problems.

Coursework will include proving theorems, deriving methods and algorithms for solving signal processing problems in vector spaces, and using Matlab to apply these methods to real-world signal processing and data science problems.

Required Texts: None.

Recommended Texts:
- D. Luenberger, *Optimization by Vector Space Methods*, Wiley, 1969.
- T. Moon and W. Sterling, *Mathematical Methods and Algorithms for Signal Processing*, Prentice Hall, 2000.
- J. A. Fessler. *Image reconstruction: Algorithms and analysis.* Unpublished book manuscript.
- C. A. Bouman. *A Guide to the Tools of Model Based Image Processing.* Unpublished book manuscript.

Grading and Course Policies:

40% Homeworks
20% Project
20% Midterm Exam
20% Final Exam

Homeworks must be turned in at the beginning of class (9:30am) on the due date. Late homeworks will receive a score of zero. The final homework grade will be based on your average score after discarding the lowest.

You are allowed (and encouraged!) to discuss homework assignments with your classmates, but are expected to complete your homework assignments individually. USC's recommended sanction for plagiarism, unauthorized collaboration, and/or cheating on any coursework is an F for the course, with a possibility for further disciplinary action.

Several of the homeworks will require MATLAB programming. It is your responsibility to make sure that you know how to access the software and read/write/debug MATLAB code.

All exams are cumulative and closed book, with no calculators. Please check now for any conflicts with the scheduled exam times.

Websites:
All course materials will be distributed through the USC Blackboard website: https://blackboard.usc.edu/. It is your responsibility to check the website regularly for updates (notes, assignments, due dates, etc.).

We will be using Piazza for class discussion. The system is highly catered to getting you help fast and efficiently from your classmates and myself. Rather than emailing questions to me, I encourage you to post your questions on Piazza so that everyone in the course can benefit from the discussion. The Piazza page for the course can be found at: https://piazza.com/usc/fall2015/ee592/home. If you have any problems or feedback for the developers, email team@piazza.com.

Suggestions:
My goal is to teach you and your classmates as much as possible about solving inverse problems, while simultaneously inspiring your interest, excitement, and curiosity about the material. This will be easier if you:

- Come to class on time and pay attention.
- Ask questions and participate in classroom discussion.
- Do all of the assignments.
- Make use of office hours.
- If you're struggling with the material, don't wait until the last minute to talk to me.
- Don't violate USC's academic integrity standards – you won't enjoy the consequences.

COURSE OUTLINE

Week 1: Inverse problems; analytic versus model-based solution approaches; least squares, maximum likelihood, penalized maximum likelihood, maximum *a posteriori*, minimum mean-squared error, and minimum absolute error estimation; linear vector spaces and subspaces.
Week 2: Linear varieties, combinations, and independence; basis; finite and infinite dimensional spaces; norms; existence and uniqueness of solutions in $\mathbb{R}^N$; left and right inverses.
Week 3: Orthogonality; projectors; fundamental theorem of linear algebra; least-squares solutions.
Week 4: Minimum norm solutions; minimum norm least-squares solutions; Moore-Penrose pseudoinverse.
Week 5: Singular value decomposition.
Week 6: Matrix norms; Eckart-Young theorem and applications; sensitivity and conditioning of $\mathbf{Ax} = \mathbf{b}$ with errors in both $\mathbf{A}$ and $\mathbf{b}$.
Week 7: Sensitivity and conditioning of minimum norm least squares and least squares problems; SVD filtering; Tikhonov regularization; total least squares and applications.
Week 8: Landweber iteration; conjugate gradient method; Gauss-Markov theorem; A-, D-, and E-optimal experiment design.
Week 9: MIDTERM; Nonlinear regularization; M-estimators and influence functions; majorize-minimize methods, expectation maximization, and iterated conditional modes.

Week 10: Sparsity-constrained inverse problems in $\mathbb{R}^N$; proofs of perfect reconstruction for ℓ_0 and ℓ_1 minimization under restricted isometry conditions.
Week 11: Constrained optimization; KKT conditions; penalty method; augmented Lagrangian method.
Week 12: Hamel bases, normed vector spaces, the $\ell_p(\mathbb{Z}^N)$ and $\mathcal{L}_p(\mathbb{R}^N)$ vector spaces.
Week 13: Equivalence classes; inner product spaces; induced norms; parallelogram law and polarization identity; linear operators; norms on linear operators; adjoints; equivalence of norms in finite dimensional spaces.
Week 14: Matrix representations of inverse problems in finite dimensional spaces; convergence of vector sequences; vector Cauchy sequences; Banach spaces; Hilbert spaces; minimum norm least squares problems in Hilbert spaces.
Week 15: VARPRO; Stochastic methods for solving energy minimization problems (Gibbs sampling, Metropolis-Hastings algorithm).
Week 16: FINAL EXAMINATION

Statement for Students with Disabilities

Any student requesting academic accommodations based on a disability is required to register with Disability Services and Programs (DSP) each semester. A letter of verification for approved accommodations can be obtained from DSP. Please be sure the letter is delivered to me (or to TA) as early in the semester as possible. DSP is located in STU 301 and is open 8:30 a.m.–5:00 p.m., Monday through Friday.

Website and contact information for DSP:
http://sait.usc.edu/academicsupport/centerprograms/dsp/home_index.html,
(213) 740-0776 (Phone), (213) 740-6948 (TDD only), (213) 740-8216 (FAX), ability@usc.edu.

Statement on Academic Integrity

USC seeks to maintain an optimal learning environment. General principles of academic honesty include the concept of respect for the intellectual property of others, the expectation that individual work will be submitted unless otherwise allowed by an instructor, and the obligations both to protect one's own academic work from misuse by others as well as to avoid using another's work as one's own. All students are expected to understand and abide by these principles. *SCampus*, the Student Guidebook, (www.usc.edu/scampus or http://scampus.usc.edu) contains the University Student Conduct Code (see University Governance, Section 11.00), while the recommended sanctions are located in Appendix A.

Emergency Preparedness/Course Continuity in a Crisis

In case of a declared emergency if travel to campus is not feasible, USC executive leadership will announce an electronic way for instructors to teach students in their residence halls or homes using a combination of Blackboard, teleconferencing, and other technologies.