
Chapter 4. Random Walks.
4.2. Recurrence. Results for simple random walk on \( d \)-dimensional integer lattice.
4.4. Renewal theory. The renewal theorem (non-arithmetic case).

Chapter 5. Martingales. (Discrete time only)
5.1. Conditional expectation (with respect to a \( \sigma \)-algebra).
5.3. Examples (including one of Polya’s urn schemes, and the Galton-Watson branching process).
5.4. Doob’s inequality. \( L^p \) inequalities. \( L^p \) martingale convergence theorems.
5.5. Uniform integrability. \( L^1 \) martingale convergence.
5.6. Backwards martingales.
5.7. Optional stopping theorems.

Chapter 6. Markov Chains. (Countable state space in 6.4, 6.5, 6.6.)
6.1. Basic definitions.
6.2 Examples (branching process, renewal chain, M/G/1 queue, Ehrenfest chain, birth and death chain).
6.4. Recurrence and transience. Irreducibility.
6.6. Asymptotic behavior (as time goes to infinity).
6.8. General state space (how to extend 6.4-6.6 material).

Chapter 7. Ergodic Theorems.
7.2. Birkhoff’s Ergodic Theorem. Statement only.

Stochastic Processes (notes handed out)
1.2. Finite-dimensional distributions. Kolmogorov’s construction.
1.3. Versions of a process. Equivalence.
1.4. Versions with continuous sample paths. Stochastic continuity.
1.5. Kolmogorov’s criterion for a version with continuous sample paths (stated without proof).
1.6. Separable and measurable processes.
1.7. Stationary processes.

Markov processes (continuous time). (notes handed out).
2.2. Construction of a Markov process from its transition probabilities.
2.4. The Markov semigroup. Feller and Feller-Dynkin processes.
2.5. The infinitesimal operator. Examples.
2.6 Hille-Yosida semigroup theory.
2.7 Kolmogorov’s forward and backward equations. Stationary density. Sub- and super-martingales.
2.8 Diffusion processes.

Chapter 8. Brownian motion.

8.1. Definition. Construction. Sample path behavior (Hölder continuous, nowhere differentiable).
8.2 Markov property.
8.3. Strong Markov property and stopping times.

Finally a brief discussion of (i) continuous time martingales involving Brownian motion, and (ii) the role of Brownian motion as an integrator in stochastic integrals.