Signaling New Product Reliability with After-Sales Service Contracts

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Prior studies on performance-based contracting (PBC) for after-sales services have highlighted its advantages over traditional resource-based contracting (RBC), when products are established and their reliability is known to all parties. We develop a game theoretic model to investigate how these insights are affected when the vendor is privately informed about the reliability of a newly developed product. A novel feature of our model is the interaction between reliability signaling (private information) and the vendor’s discretionary investment in spares inventory (private action), which arises naturally in the setting we consider. We find that this interaction leads to contrasting equilibrium outcomes under the two contracts: RBC induces the vendor to focus on inventory savings, leading to underinvestment in spares, while PBC induces the vendor to focus on reliability signaling, achieved through overinvestment in inventory. As a result, neither contract is efficient. We investigate two means to mitigate this inefficiency, but either approach has caveats: (a) making inventory verifiable removes the trade-off between reliability signaling and inventory investment, but results in diverging contract preferences between the vendor and the buyer; (b) pooling inventories across multiple buyers saves inventory costs but it also hinders reliability signaling, potentially exacerbating inefficiency.

Key words: signaling games; performance-based contracting; aerospace sector; after-sales services

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1. Introduction

After-sales product support, namely maintenance, repair, and overhaul (MRO) operations, is a key business area in the aerospace industry. This sector alone generated revenue of $43 billion in 2009 (Standard and Poor’s 2011). One of the main drivers of financial performance in this sector is product reliability, since MRO operations revolve around preventing and responding to flight disruptions that occur due to unanticipated product malfunctions. To mitigate the impact of imperfect reliability, it is critical for the product support provider to invest in spares inventory, since spares replace defective products and thus increase system usage. In a typical decentralized supply chain consisting of buyer organizations (e.g., airline companies) and vendors (e.g., aircraft engine manufacturers), an important challenge is to establish a contractual relationship that enables effective management of reliability and inventory.
Prior studies have emphasized the role of reliability and spares inventory in determining a firm’s payoff under two types of contract that are widely used in practice: resource-based contracts (RBC) and performance-based contracts (PBC). Under RBC, the compensation for the vendor who provides product support is proportional to the amount of resources utilized (e.g., labor and spare parts consumed to repair a defective product). By contrast, PBC unties resource usage from compensation, since the vendor under PBC is rewarded or penalized based on the realized performance outcome that directly impacts the buyer, such as aircraft up-time. A general consensus from the literature on PBC (see, for example, Hypko et al. 2010, Kim et al. (2007), Randall et al. 2011, Ward and Graves 2007) is that PBC is a superior contracting mechanism that better aligns incentives between the buyer and the vendor, resulting in higher product utilization at a lower overall cost.

Despite the consensus in the academic literature touting the advantages of PBC over RBC, practitioners appear to be ambivalent about the choice between the two, some preferring PBC based on the aforementioned merits while others remain unwilling to switch from the more traditional RBC. Such reluctance to switch from RBC may simply reflect the reality that some buyers are comfortable with the status quo. However, this inertia-based argument is not entirely consistent with some of the observed facts. In particular, it does not explain why buyers are often willing to adopt PBC when they consider acquiring products with mature technologies, but not when they acquire products equipped with newly developed technology; these differing preferences based on the maturity of the underlying technology are reported in the survey of buyer organizations conducted by the U.S. Government Accountability Office (GAO 2004). The report indicates that buyers are reluctant to switch to PBC for newly developed products due to the absence of publicly available baseline data on product reliability. In other words, buyers view RBC as offering better value than PBC when product reliability is unknown to them.

This observation identifies an important aspect of after-sales service contracting that has been overlooked in the literature: buyers’ uncertainty about product reliability and its impact on contract performance. A vendor faces a challenge in this environment because the lack of independently verifiable data on reliability limits her ability to propose the best contract terms for herself and the supply chain. That is, the vendor who bundles an after-sales service contract with a new product has to consider not only how her profit is maximized in the long run but also how she can credibly

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1 Conklin & deDecker (https://www.conklindd.com/Page.aspx?cid=1066) report, “[PBC] programmes generate a lot of heated debate, with some believing they can get their engine maintenance for a lot less money [through programmes other than PBC]. Others are convinced that PBC is the best thing since sliced bread. Still others provide a full range of opinions in between.” Responding to such heterogeneous preferences, vendors have adopted different priorities for their contract offerings. For example, Rolls-Royce reports that in the period 2001–2009, 80% of its large commercial-airliner engines (the Trent 700 and its variants) were covered by PBC, and this is forecast to grow in the future. At the same time, at Pratt & Whitney only a third of the installed engine base in 2009 was covered by PBC and the company is reporting a slowdown in PBC agreements going into the future (Adler et al. (2009)).
Resource-based contract | Performance-based contract
---|---
Verifiable inventory | §5.1 Efficient reliability signaling and inventory investment, with partial rent extraction
| §5.2 Efficient reliability signaling and inventory investment, with full rent extraction
Unverifiable inventory | §6.1 Focus on inventory savings over reliability signaling; underinvestment in inventory
| §6.2 Focus on reliability signaling over inventory savings; overinvestment in inventory

Table 1 Summary of main results

We develop a stylized principal–agent model that captures the dynamics arising when a vendor (principal) possesses an informational advantage concerning the reliability of a newly developed product that a buyer (agent) plans to acquire. We adopt the classical signaling framework as the model basis, but with an important added feature that makes the dynamics richer and more subtle: an interaction between the vendor’s reliability signaling and her discretionary inventory investment. While investing in spares inventory compensates for imperfect reliability (the more the spares, the smaller the chance of prolonged outage after a product failure), this mitigating effect also interferes with the vendor’s ability to credibly signal product reliability through contract terms. This interaction forces the vendor to set contract terms so that the dual goals of inventory cost savings and effective signaling are balanced.

Our analysis reveals how RBC and PBC differ as vehicles to signal product reliability. As it turns out, verifiability of inventory plays a crucial role. When inventory is verifiable, both RBC and PBC are effective as a signaling mechanism as they allow the vendor to credibly convey private reliability information and achieve supply-chain efficiency. When inventory is unverifiable, by contrast, neither contract brings efficiency. In particular, the two contracts shift the balance between reliability signaling and inventory investment in opposite directions: RBC shifts the focus to inventory savings over effective signaling, whereas PBC shifts focus to effective signaling over inventory savings. Consequently, the supply chain is left with underinvestment in inventory under RBC and overinvestment in inventory under PBC. Table 1 summarizes these findings.

This observation suggests that the two supply-chain parties may have a mutual interest in having the vendor invest in technology to make inventory verifiable, as doing so will decouple the interaction between reliability signaling and inventory investment and bring supply-chain efficiency, thereby potentially benefitting both parties. We find, however, that allowing for such technology investment results in diverging contract preferences between the vendor and the buyer; the vendor...
prefers PBC, whereas the buyer prefers RBC. Thus, the prediction from our analysis is consistent with the aforementioned reports from the aerospace industry. Another option for improving supply-chain efficiency, which is of particular interest when inventory verifiability is prohibitively expensive, is for the vendor to pool inventory across multiple buyers. We find that this option also comes with limitations due to its interaction with signaling; while pooling saves inventory costs, it hinders reliability signaling, potentially exacerbating inefficiency. Overall, our findings show that after-sales service contracting for newly developed products presents new set of challenges that require managerial actions that are quite distinct from those used for established products.

The paper is organized as follows. After a literature review in §2, we present our model in §3 followed by a brief discussion of the benchmark first-best scenario in §4. To illustrate the structural differences between RBC and PBC, we conduct our analysis in §§5–6 assuming that the contract type is exogenously specified. In §5 we focus on the problem of signaling in isolation, assuming that inventory is verifiable and is contracted upon. In §6 we study how the vendor’s discretionary inventory choice interacts with her reliability signaling. After evaluating how RBC and PBC perform under these scenarios, we discuss in §7 the two means by which system inefficiency can be alleviated: (a) contract choice coupled with investment in inventory verifiability, and (b) inventory pooling. We conclude in §8.

2. Literature Review

Our work relates to three distinct streams of literature: the first explicitly deals with PBC for outsourced services; the second, with which our work shares a methodological connection, is the literature on asymmetric information in operations management (OM); and the third is the literature on the use of warranties as a signaling mechanism.

PBC has been studied in a service delivery setting by, among others, Roels et al. (2010), Gumus et al. (2011), and Jain et al. (2013). PBC in the context of after-sales services in the aerospace industry has been studied in Kim et al. (2007) and Kim et al. (2010). Our paper is closest in spirit to Kim et al. (2007), who investigate a setting in which the buyer proposes the terms of the PBC while the vendor exerts private effort to reduce maintenance cost and invests in spare parts inventory. The paper shows that in a setting with risk-averse players the first-best cannot be attained and the optimal second-best contract involves a performance-related component. While this stream of research has shown that PBC is a preferred contracting mechanism in settings under moral hazard issues because of its ability to align incentives between the vendor and buyer, it invariably assumes that the failure characteristics of the products are common knowledge. Our paper complements this line of research by studying a setting with asymmetric information: the vendor has better knowledge about the product’s reliability than the buyer does because the product is newly developed. In
contrast to previous studies, we identify circumstances under which PBC is not favored due to the presence of asymmetric information.

Our paper is related methodologically to the OM literature on games of asymmetric information. Examples of screening games, where the uninformed principal offers contracts designed to elicit information from the informed agent, include Corbett et al. (2004) and Li and Debo (2009). Our work is closer to papers that adopt a signaling framework (e.g., Anand and Goyal 2009, Cachon and Lariviere 2001), in which an informed principal signals her superior information through the contracts offered to an uninformed agent. We contribute to this literature by studying how PBC and RBC perform as signaling devices. Unlike the aforementioned works, which implicitly assume that the contracting parties somehow commit not to renegotiate, in our analysis we employ the signaling-with-renegotiation framework developed by Beaudry and Poitevin (1993), where the agents may renegotiate after a contract has been signed but before it is implemented. (It can be shown using this framework that the classic (and inefficient) second-best equilibrium outcome of a one-shot signaling game that ignores renegotiation (for example, Theorem 7 in Cachon and Lariviere (2001)) is never renegotiation proof and that the renegotiation-proof outcome is, in general, more efficient.)

Correspondingly, our work also complements the recent OM literature on the impact of renegotiation on equilibrium outcomes (for example, Plambeck and Taylor (2007b), Plambeck and Taylor (2007a), Xiao and Xu (2012)). While in these articles renegotiation is triggered when some of the uncertainty is naturally resolved after the contract is implemented, in our setting renegotiation is triggered by the potential for Pareto improving offers that arises at the time a contract is signed but before the contract is actually implemented.

The risk-sharing and signaling capabilities of RBC and PBC result in characteristics similar to those in product warranties. Research on signaling product quality through warranties has a long tradition in economics (Gal-Or 1989, Lutz 1989, Riley 2001), marketing (Boulding and Kirmani 1993, Moorthy and Srinivasan 1995), and OM (Courville and Hausman 1979, Gumus et al. 2011). See Kirmani and Rao (2000) for an extensive survey of the literature. The underlying premise of this literature is that firms selling low-quality products will face higher costs for the same level of warranty than will high-quality firms because low-quality firms’ products are likely to require more frequent repair. A major difference between our work and those in the quality-signaling literature is that our results are driven by the interaction between the principal’s private information (about product reliability) and private action (represented by inventory investment), a feature that gives rise to new dynamics. Another departure from the extant literature is that, in addition to identifying the scope for signaling, our model allows us to evaluate the performances of two widely-used contracts in the after-sales setting: RBC and PBC.

While many papers in the signaling literature have investigated the interaction between a principal’s private information and an agent’s moral hazard, the interaction between a principal’s private information and her own private action has received little attention. To the best of our knowledge, the only other paper that has studied this type of...
3. Model

We consider a supply chain that consists of two risk-neutral parties: a vendor (“she”) and a buyer (“he”). The buyer faces constant demand for product usage that he plans to satisfy by purchasing and deploying $N$ identical copies of a newly developed product from the vendor. The vendor provides after-sales repair and maintenance services to the buyer. The vendor sets the terms of the service contract and proposes them to the buyer. Hence, the vendor serves as a principal in the principal–agent framework. The contract takes one of two forms: RBC or PBC. The duration of the contracting period is normalized to one.

3.1. Repair Process and Inventory Policy

We adopt the standard modeling framework established in the literature to represent the repair process and spares inventory management, as elaborated by Kim et al. (2007, 2011), among others. During the course of deployment the products fail occasionally due to malfunction. These products are repairable items that are not discarded upon failure but repaired and restored to working order. We focus on the types of failure that require unscheduled repairs, which incur large and unanticipated costs. For instance, unscheduled maintenance on a wide-bodied aircraft could cost as much as ten times the capital outlay over the operating life of the aircraft (see Hopper (1998)). We do not explicitly model preventive and scheduled maintenance, the cost of which is either fixed or regulated and is not significantly affected by contractual incentives.

A one-for-one base stock policy is employed for spares inventory control (Feeney and Sherbrooke 1966). A failed unit immediately enters a repair facility, which is modeled as a $GI/G/\infty$ queue with expected repair lead time $l$. We assume that the distribution for repair lead time is exogenously specified. The expected number of product failures during the contracting period is denoted by $\mu$. We assume that the arrival process for failures is exogenous and state-independent: a common convention found in the spare parts inventory management literature (e.g., Sherbrooke 1968). Although in practice there are situations where this condition is violated, it is not overly restrictive as the infrequent nature of failures makes this an excellent approximation.

A product failure may affect system availability, which is defined as the fraction of system up-time over the length of contract duration. System availability is unaffected if a spare product can be pulled from the inventory to replace the defective unit immediately. If the inventory is empty at the time of failure, however, system availability is reduced until a repaired unit becomes available from the repair facility; this causes inventory backorder, denoted by $B$. It is clear from this description interaction is Jost (1996). In this paper, however, the authors focus on a completely different setting in which the principal chooses from a binary action set and the agent’s payoff is independent of this action. As a consequence of these differing assumptions, they obtain results that are very different from ours: they predict that only a pooling equilibrium will emerge.
that backorders are reduced as the inventory \( s \) increases. In our analysis we approximate all discrete variables (including \( B \) and \( s \)) as continuous variables in order to facilitate game-theoretic analysis.

Let \( F \) be the stationary distribution function of the inventory on order \( O \), that is, the number of repairs being performed at the repair facility at a given point in time. When a one-for-one base stock policy is followed, \( O \) can be thought of as the number of busy servers in a \( GI/G/\infty \) queue, the distribution for which is stationary for any finite repair lead time \( l \) (Kaplan 1975). We assume that \( F(x) = 0 \) for \( x < 0 \) and the corresponding density function \( f(x) > 0 \) for \( x > 0 \). We also assume that the on-order distribution has the increasing hazard rate property, i.e., \( \frac{f(x)}{1-F(x)} \) is monotone increasing in \( x \). This property is satisfied by a wide range of distributions including Gamma, Weibull, Poisson, and truncated normal (Barlow et al. 1963, Gupta et al. 1997). For a given level of spares inventory \( s \), the expected backorders in steady state is then equal to \( E[B|s] = \int_s^\infty (1-F(x))dx \). Moreover, a one-to-one correspondence can be made between system availability and the expected backorders: availability is equal to \( 1 - E[B|s]/N \). For this reason, we use the terms “availability” and “backorders” interchangeably throughout the paper.

### 3.2. Cost Structure

Given the assumption that the buyer faces constant demand for product usage, the maximum revenue that she can generate (in case there is no product outage) is a constant. Without loss of generality, we normalize this value to zero. Each time a product failure occurs, the buyer incurs a fixed cost \( r \geq 0 \). In commercial airline operations, \( r \) represents, among other things, the cost associated with rescheduling flights that results from delays due to an engine coming off-wing for repair (Adamides et al. 2004). In addition, if the vendor does not have inventory on hand to replace the failed product, the buyer incurs a variable cost \( \chi > 0 \) per unit time until the system function is restored. The cost \( \chi \) represents the direct revenue loss to the buyer resulting from system inoperability due to the failed product (for example, the cost of grounding an aircraft). The buyer’s expected cost is equal to \( \mu r + \chi E[B|s] \).

The vendor expects to incur a repair cost (such as the cost of labor and component parts) equal to \( M \) after each failure, plus the cost of acquiring and maintaining spares inventory, each unit costing \( c \) per unit time. Note that since we have normalized the duration of the contract to 1, the costs per unit for the whole duration of the contract are also \( c \). The vendor’s expected cost is equal to \( \mu M + cs \). To rule out trivial results, we assume \( \chi \geq c \).

### 3.3. Information Structure

As the developer of a product equipped with new technology, the vendor possesses superior knowledge about the product’s characteristics, including an estimate of its failure distribution, at the time she introduces the product to the market (GAO 2004, Boito et al. 2009). While this gives the
vendor an informational advantage, it also creates an incentive to misrepresent the information: a vendor whose product has low reliability may claim otherwise. This presents a challenge for the vendor whose product has high reliability, since any such claim may not be viewed by the buyer as credible. This problem is compounded by the fact that the buyer lacks the ability to independently verify the vendor’s claim. For example, Kappas (2002) reports that OEMs’ databases describing the material properties of alloys in aircraft engines are not publicly available. More importantly, inferring the reliability of products from actual failures of deployed units is challenging because little historical data exist for the new technology and failures occur infrequently; for instance, the median time between failures for an aircraft engine reported in Guajardo et al. (2012) is five years.

To represent information asymmetry in a succinct and analytically tractable way, we assume that the product offered by the vendor is one of two possible types, $L$ or $H$, where $L$ denotes low reliability and $H$ denotes high reliability. We use the terms “unreliable vendor” and “reliable vendor” to refer to the vendor having a low reliability product type and high reliability product type, respectively. While the vendor observes her type perfectly, the buyer believes ex-ante that the vendor is of type $L$ with probability $p > 0$. This belief is common knowledge. The expected number of product failures occurring during the contract period satisfies $\mu_L > \mu_H$, i.e., more failures are expected to occur when product reliability is low. Moreover, the distributions $F_L$ and $F_H$ of the on-order inventory $O_L$ and $O_H$ are assumed to have the following two properties:

$$\frac{f_L(x)}{1 - F_L(x)} > \frac{f_H(x)}{1 - F_H(x)},$$

$$\int_{F_H^{1-p}(x)}^{\infty} (1 - F_H(x)) \, dx \leq \int_{F_L^{1-p}(x)}^{\infty} (1 - F_L(x)) \, dx, \quad \forall p \in [0, 1].$$

In other words, the distribution functions follow the hazard rate ordering (1) and the excess wealth (EW) ordering (2) (see Shaked and Shanthikumar (2007) for details). It can be shown that these properties are satisfied if the failures occur according to a Poisson process. We note that (1) and (2) play important roles in our analysis in identifying the equilibria.

In order to focus on the dynamics arising from signaling we sidestep the issue of double moral hazard, i.e., the possibility that the buyer exerts insufficient level of care for the product he has acquired (Lutz 1989, Jain et al. 2013). This modeling choice is reasonable given that our research motivation comes primarily from the aerospace industry, where vendors typically have real-time access to information about the usage patterns and performance of their deployed products, thereby alleviating concerns about buyer moral hazard. Furthermore, we do not model risk aversion explicitly as it would add another layer of complexity, diverting attention away from the main trade-offs we investigate.
3.4. Contract Types and Payoffs

In this paper we restrict attention to two predominant types of maintenance contracts used in the aerospace industry: resource-based contract (RBC) and performance-based contract (PBC). As evidenced by numerous industry reports (e.g., Adler et al. 2009), practitioners focus on the contrasts between these two types especially since growing adoption of PBC in recent years. The two contract types differ on the basis of compensation for maintenance activities performed by the vendor. RBC is the older and more traditional transaction-based approach, which includes the popular time and material (T&M) contract. It is based on the simple idea that compensation for the vendor is proportional to the amount of resources utilized to repair a defective product, such as labor and spare parts consumption. In its general form, RBC also includes warranty coverage that is designed to protect the buyer from any unexpected out-of-pocket expenses incurred after a product failure. In this paper we generalize the definition of RBC to include warranty coverage in addition to compensation for utilized resource. PBC is a fundamentally different concept in that the vendor is compensated based on the realized performance outcome (for example, aircraft up-time) instead of the amount of resources utilized for repairs. A typical PBC specifies the rate at which a vendor will be paid per unit of time the product is functional or, equivalently, the rate at which the vendor will be penalized for each unit of product downtime.

Formally, a contract \( C \) is a vector of real-valued parameters that specifies the total transfer payment made by the buyer to the vendor. Depending on whether or not the inventory \( s \) is verifiable, \( C \) may include \( s \), in addition to other contract parameters. RBC is characterized by the fixed fee \( w \geq 0 \) and the warranty coverage \( \alpha \), which assumes a value between 0 and 1 that represents the buyer’s share of the repair cost incurred after a product failure. Thus, \( \alpha = 0 \) denotes full warranty coverage (i.e., the vendor bears the entire repair cost), whereas \( \alpha = 1 \) represents no warranty (i.e., the buyer is responsible for the entire cost). RBC is then represented as either \( C = (w, \alpha, s) \) or \( C = (w, \alpha) \), and the expected transfer payment is equal to \( T(C) = w + \alpha \mu M \). On the other hand, PBC is characterized by the fixed fee \( w \geq 0 \) and the performance penalty \( v \geq 0 \) charged to the vendor for each unit of product downtime. Therefore, PBC is represented as either \( C = (w, v, s) \) or \( C = (w, v) \), and the expected transfer payment is equal to \( T(C) = w - v \mathbb{E}[B|s] \).³

With the transfer payments defined as above we can write the buyer’s expected payoff as

\[
U(C) = -T(C) - \chi \mathbb{E}[B|s] - \mu r;
\]

similarly, the vendor’s expected payoff is given by

\[
V(C) = T(C) - cs - \mu M.
\]

³ Notice that neither contract penalizes the vendor based on the number of product failures. Contracts that penalize the vendor based on the number of failures are rarely observed in practice, as vendor firms may invoke force majeure claiming that random failures are beyond their control. Both RBC and PBC provide contractual remedies for post-failure damages, rather than preventive protections.
3.5. Signaling Game

Consistent with the majority of signaling models found in the literature, we assume that the vendor, who possesses private information about the reliability of a newly developed product, chooses contract terms and proposes them to the buyer. The buyer then accepts the contract terms if his expected payoff exceeds an outside option valued at $\theta$. In §5 and §6 we present our analysis under the premise that the contract type is exogenously given (either RBC or PBC). This is an assumption we relax in §7.1, where we consider endogenous contract choice.

We seek a perfect Bayesian equilibrium (PBE) that is also renegotiation-proof. When a PBE is inefficient (e.g., second best) it is potentially vulnerable to Pareto-improving renegotiations after it is signed but before it is executed; for such cases we adopt the signaling-with-renegotiation framework developed by Beaudry and Poitevin (1993). As is common in signaling games, we are confronted with the issue of multiple equilibria, which hinders the ability to make sharp predictions about the equilibrium outcome. To circumvent this multiplicity problem we also require the candidate PBEs to satisfy the Extended Divinity (XD) criterion. This refinement captures the notions behind both the Divinity Criterion (Banks and Sobel 1987) and the Intuitive Criterion (Cho and Kreps 1987), suitably adapted to a multi-period treatment to accommodate renegotiation-proofness (see Beaudry and Poitevin (1993) for more details). As it turns out, the XD criterion identifies a set of equilibria that are payoff-equivalent; i.e., the surviving equilibria are unique with respect to the payoff, even though the combinations of contract terms that lead to the outcomes may differ.

With the equilibrium identification strategy set, we outline the sequence of events as follows.

**Step 1.** Nature reveals to the vendor her type $\tau \in \{L, H\}$.

**Step 2.** The vendor offers contract terms (possibly type-contingent) to the buyer.

**Step 3.** The buyer updates his beliefs about the vendor’s type and accepts or rejects the contract. If the buyer rejects the contract, the game ends. If he accepts and the resulting outcome is efficient, the negotiation process is terminated (since Pareto-improving renegotiations do not exist) and we proceed directly to Step 5.

**Step 4.** The outcome in Step 3 is renegotiated as per Beaudry and Poitevin (1993).

**Step 5.** The vendor decides the inventory of spares to be maintained (if not specified in the contract); products are deployed, failures occur, and repair and maintenance takes place; transfer payment is made by the buyer and final payoffs are realized by both players.

4. Benchmark: First-Best Under Complete Information

We first establish the first-best benchmark against which the performances of different contractual agreements are to be compared. The first-best treatment requires that all attributes, decisions, and actions are completely observable and verifiable to both parties. Under such a condition, it is
sufficient to devise a contract consisting only of the fixed fee $w$ and the inventory $s$; no warranty (as under RBC) or performance penalty (as under PBC) term is needed. Given the vendor type $\tau \in \{L, H\}$, the first-best outcome is obtained by solving the optimization problem

$$\max_{w, s \geq 0} V_\tau = w - cs - \mu_\tau M,$$

subject to $U_\tau = -w - \chi E_\tau(B|s) - \mu_\tau r \geq \theta$. \textit{(IR)}

The solution is summarized in the following proposition.

**Proposition 1.** When the vendor’s type $\tau \in \{L, H\}$ and inventory are verifiable, the optimal contract specifies the first-best contract parameters $\bar{s}_\tau = F^{-1}_\tau(1 - c/\chi)$ and $\bar{w}_\tau = -\chi E_\tau(B|\bar{s}_\tau) - \mu_\tau r - \theta$. Furthermore, the reliable vendor achieves a higher payoff and maintains a lower inventory than the unreliable vendor: $V_H > V_L$; $\bar{s}_H < \bar{s}_L$.

All proofs are found in the Appendix. The result $V_H > V_L$ confirms our intuition that the reliable vendor is better off than the unreliable vendor when no information asymmetry exists. The next result, $\bar{s}_H < \bar{s}_L$, states that the optimal inventory of the reliable vendor is lower than that of the unreliable vendor. This is a direct consequence of the substitutable relationship between reliability and inventory: the higher the reliability, the less frequently product failures occur and, therefore, the required level of spares inventory is lower. As we discuss in the subsequent sections, this inherent dependence between reliability and inventory has significant implications for signaling effectiveness when information is asymmetric.

## 5. Signaling With Verifiable Inventory

We now consider the setting in which the vendor possesses private information about product reliability. Note that the reliable vendor’s lower inventory and higher payoff in the benchmark case together imply that the first-best contracts are not incentive compatible; i.e., the unreliable vendor has an incentive to misrepresent her type. We first examine the case in which the vendor’s inventory choice can be verified and hence included as part of the contract terms. This analysis allows us to isolate the effect of signaling under RBC and PBC and establish a baseline for the next case presented in §6: a more complex scenario in which the vendor makes a discretionary choice of inventory in addition to her signaling effort.

### 5.1. Resource-Based Contract

When inventory is verifiable the vendor includes it in her RBC contract along with other parameters. Thus, the contract specifies the inventory $s_\tau$, the fixed fee $w_\tau$, and the warranty coverage $\alpha_\tau \in [0, 1]$ that defines the buyer’s share of the repair cost. These terms depend on the vendor type $\tau \in \{L, H\}$. Under the assumptions laid out in §3, the payoff of the vendor of type $\tau$ is equal
to $V_\tau = w_\tau - (1 - \alpha_\tau)\mu_\tau M - cs_\tau$, while the payoff of the buyer, when the vendor is of type $\tau$, is $U_\tau = -w_\tau - \alpha_\tau\mu_\tau M - \chi\mathbb{E}_\tau(B|s_\tau) - \mu_\tau r$.

Since the warranty coverage offers to protect the buyer from unanticipated out-of-pocket expenses incurred after a product failure, the value of $\alpha_\tau$ set by the vendor in her contract may relay information about whether or not the product is reliable. The question is: Can the reliable vendor credibly signal her type using the warranty term included in RBC? The following proposition answers this question. Throughout the paper we use the superscript * to denote the equilibrium outcomes.

**Proposition 2.** When inventory is verifiable, the PBEs of a signaling game under RBC that satisfy the conditions in §3.5 are payoff-equivalent, separating, and efficient. In each equilibrium, $s^*_L > s^*_H$, $w^*_L < w^*_H < \overline{w}$, and $\alpha^*_L \geq \alpha^*_H = 0$. As a result, the unreliable vendor recovers her first-best payoff but the reliable vendor does not: $V^*_L < V^*_H < \overline{V}$. The buyer is left with a positive expected rent above his outside option $\theta$.

Although we find that multiple equilibria exist, they are all payoff-equivalent; i.e., they only differ in the amount of warranty and fixed fee offered by the unreliable vendor without altering the net payoff. In all equilibria the reliable vendor signals her type (i.e., they are separating equilibria) by setting a lower inventory than the unreliable type ($s^*_L > s^*_H$) and offering a full warranty ($\alpha^*_H = 0$) in return for a higher (but discounted from first-best) fixed fee ($w^*_L < w^*_H < \overline{w}$). This ability to differentiate herself from the unreliable type comes at a cost, as she fails to earn the same amount of payoff that she would have earned under the first-best condition ($V^*_H < \overline{V}$). It is worth noting that this reduction in payoff for the reliable vendor does not in fact result in system inefficiency; at the supply-chain level no distortion exists, as evidenced by the same inventory investments as under the first-best ($s^*_L = \overline{s}_L, s^*_H = \overline{s}_H$). Instead the reduction in payoff of the vendor is captured entirely by the buyer.

Two aspects in Proposition 2 warrant further examination. First, in order to understand how RBC allows the vendor to signal, it is instructive to examine the trade-offs that the reliable vendor faces. She has a dual objective: maximize her payoff while signaling her type by differentiating herself from the unreliable type. Maximizing profit can be further decomposed into “enlarging the pie size” and “dividing the pie.” Among the three levers at the vendor’s disposal – the inventory $s_H$, warranty coverage $\alpha_H$, and fixed fee $w_H$ – only the inventory has an impact on the pie size; i.e., on supply-chain efficiency. This is because higher inventory increases product availability and hence determines system performance. The fixed fee and warranty do not impact efficiency since they represent transfers within the supply chain. The former is used by the vendor to extract rents from the buyer. The reason the warranty term is used as a primary signaling device is because the
marginal impact of increasing warranty differs between the two vendor types, whereas the marginal impact of altering the fixed fee or inventory is exactly the same for the two. Specifically, since high warranty coverage requires a vendor to internalize more repair costs, it is more costly for the unreliable vendor to match the same coverage offered by the reliable vendor, who enjoys less frequent product failures.

A second feature of the equilibrium outcome with RBC is that it leaves the buyer with surplus without sacrificing efficiency. This feature distinguishes this equilibrium from the classic second-best separating equilibrium – for example, in the job market signaling analysis (Spence 1973) – in which economic value is destroyed without leaving any surplus to the agent. The reason for this distinction is that, unlike Spence’s example, in which the employee’s education does not impact the employer’s payoff, the warranty offered under RBC directly increases the buyer’s payoff. Thus, the vendor’s investment in the signal, represented by higher warranty coverage, is captured by the buyer. Moreover, the requirement that the equilibrium contract should be immune to Pareto-improving renegotiations pushes the outcome towards efficiency.

In summary, successful signaling by the reliable vendor is possible under RBC when her inventory choice is verifiable, but it comes at a cost since the vendor has to leave positive surplus to the buyer.

5.2. Performance-Based Contract

With inventory verifiable, a PBC contract specifies the inventory $s_\tau$, fixed fee $w_\tau$, and penalty rate $v_\tau$ applied to each unit of downtime (or equivalently, backorder). The contract terms may differ by vendor type $\tau \in \{L,H\}$. The payoff for the vendor of type $\tau$ is $V_\tau = w_\tau - v_\tau E(\alpha|s_\tau) - c s_\tau - \mu_\tau M$, while the payoff for the buyer is $U_\tau = -w_\tau + (v_\tau - \chi) E(\alpha|s_\tau) - \mu_\tau r$. The next result parallels Proposition 2 obtained for the RBC case.

**Proposition 3.** When inventory is verifiable, the PBEs of a signaling game under PBC that satisfy the conditions in §3.5 are payoff-equivalent, separating, and efficient. In each equilibrium, $s_L^* > s_H^* = \bar{s}_H$ and $w_L^* < w_H^*$. The penalty rates $v_H^*$ and $v_L^*$ satisfy the relation $v_L^* [E_L(B|s_L^*) - E_H(B|\bar{s}_L)] \leq \chi [E_L(B|\bar{s}_L) - E_H(B|\bar{s}_H)] + (\mu_L - \mu_H)r + c(\bar{s}_L - \bar{s}_H) \leq v_H^* [E_L(B|\bar{s}_H) - E_H(B|\bar{s}_H)]$. Moreover, both vendor types recover their first-best payoffs: $V_L^* = V_H^* < V_L = V_H$. The buyer is left with his outside option $\theta$.

Recall that the warranty term $\alpha_\tau$ served as the primary signaling device under RBC. Under PBC, the same role is played by the penalty term $v_\tau$. Similar to the RBC case, the multiple equilibria identified in Proposition 3 for the PBC case differ only in their fixed fee/penalty combinations and do not affect the vendor’s net payoff. Also similar to the RBC case is the result that the first-best inventory levels are maintained ($s_L^* = \bar{s}_L, s_H^* = \bar{s}_H$).
Despite the similarities, there are a few significant departures from the RBC case. First, unlike under RBC, the reliable vendor under PBC signals her type without ceding rents to the buyer. Instead, she retains the entire surplus and attains her first-best payoff ($V^*_H = \bar{V}_H$). Second, despite the seemingly conflicting goals of successfully signaling her type while maximizing profit, the reliable vendor is able to achieve separation from the unreliable vendor with a wide range of penalty rates. This is evidenced by the condition for $v^*_L$ and $v^*_H$ given in Proposition 3, which imposes loose constraints on the two values, even including the case in which the reliable vendor subjects herself to a lower penalty rate than the unreliable type would.\(^4\) Combined, these observations suggest that, from the vendor’s perspective, PBC is superior to RBC as a signaling device since it permits the reliable vendor to retain all of her rents.

What is different about PBC that makes it a superior signaling mechanism? To answer this, it is important to recognize that the types of risk that are shared between the vendor and the buyer under RBC and PBC are not the same. Under RBC, it is the repair risk that is shared through the warranty coverage; the vendor performs a costly repair each time a random product failure occurs, and the contract specifies how much of this cost is reimbursed by the buyer. By contrast, under PBC, it is the outage risk that is shared through the penalty term; the buyer incurs a cost proportional to the outage duration after each failure (e.g., lost revenue), and the contract specifies to what degree this cost is compensated for by the vendor. While the repair risk is a function of reliability only, the outage risk is a function of reliability as well as inventory choice, since higher inventory lowers the expected outage duration. The limitation of RBC then becomes clear: even if warranty is used to its maximum extent (i.e., full coverage is offered) it still exposes the buyer to the risk of prolonged outage because the warranty does not cover this risk. PBC relaxes this constraint because it ties the vendor’s compensation directly to the outage risk that the buyer cares about. This risk-sharing arrangement allows the vendor to signal her type more efficiently.

As noted in §2, PBC is known to align the incentives better than RBC when efficiency loss due to decentralization is caused by private actions. In the context of our problem, in which private information plays the central role, the same characteristic of PBC manifests itself as a signaling advantage.

Therefore, successful signaling by the reliable vendor can be done under both RBC and PBC when the inventory is verifiable. However, PBC presents an advantage because its risk sharing mechanism is more amenable to signaling than that of RBC, thus allowing the reliable vendor to extract all buyer surplus without compromising efficiency.

\(^4\) The reason for such a wide range of permissible penalty rates is that, under PBC with unverifiable inventory, the incentive compatibility constraints required for separation are not binding in equilibrium, even though the first-best payoffs are attained. This is one of the three qualitatively different equilibrium outcomes possible in signaling games, the other two being the second-best separating equilibrium and the pooling equilibrium (Spence 2002).
6. Signaling With Unverifiable Inventory

In this section we relax the assumption that inventory is verifiable. We examine how the two sources of inefficiency (absence of verifiability and private information) interact with each other and whether RBC and PBC lead to qualitatively different outcomes. In this setting, inventory cannot be included in a contract and, therefore, the vendor cannot use it to relay information about product reliability. As a result, the vendor is more limited in her ability to signal, but at the same time she has more freedom to set the inventory to a level that would improve her payoff. The vendor is then faced with the challenge of balancing the potential benefit of payoff increase with the potential downside of signaling ineffectiveness. We study these issues in this section.

6.1. Resource-Based Contract

Even with inventory no longer verifiable, the payoff functions for the vendor and the buyer under RBC remain the same as those of the verifiable inventory case. The difference now is that the vendor of type $\tau \in \{L, H\}$ makes a discretionary choice about the inventory $s_\tau$ that will indirectly influence the choices of the other two variables that are included in the RBC contract: the fixed fee $w_\tau$ and the warranty coverage $\alpha_\tau$. We summarize the equilibrium of the signaling game for this case in the next proposition.

**Proposition 4.** When inventory is unverifiable, the PBE of the signaling game with RBC, which satisfy the conditions in §3.5, are payoff-equivalent and inefficient, with $s_L^* = s_H^* = 0$, $\alpha_H^* = 0$ and $w_H^* = w_L^* + \alpha_L^* \mu_L M$. The PBE can be either pooling or separating, but the reliable vendor cannot recover her first-best payoff: $V_H^* < V_H^\ast$. The buyer’s payoff is equal to his outside option $\theta$.

Recall from Proposition 2 that when inventory is verifiable and hence is included in a contract, the reliable vendor using RBC succeeds in signaling her type by offering full warranty coverage combined with a discounted fixed fee, maintaining the first-best inventory. This characterization stands in direct contrast to that of the equilibrium identified in Proposition 4. With contracting on inventory no longer an option, even though a separation from the unreliable vendor can be achieved, separation does not bring any benefit over the pooling equilibrium, in which both types of vendor offer full warranty ($\alpha_L^* = \alpha_H^* = 0$) and set the inventory level to zero ($s_L^* = s_H^* = 0$). In other words, none of the combinations of contractual levers at her disposal allow the vendor to generate a payoff greater than that of the pooling outcome.\(^5\)

Intuitively, this happens because the vendor’s inability to make inventory verifiable removes any benefit that it might have brought the vendor. Inventory is costly to keep, so it is best for the

\(^5\) Maintaining zero inventory is a by-product of the simplifying assumptions of our model that allow us to focus on the main trade-offs. In reality, the vendor may choose to keep some minimum acceptable level of inventory $s > 0$ for reasons that are not captured in the model, such as the reputation effect. Our analysis indicates that, as long as this minimum level $s$ is no greater than a threshold $s^* < \bar{s}_H$, none of the qualitative insights of this paper are impacted.
vendor to minimize its level. Given that inventory cannot be used as a signaling device and that RBC does not incentivize the vendor to mitigate the outage risk with inventory, setting \( s_r \) to its minimum value is indeed the optimal course of action for the vendor. The vendor then adjusts other contract parameters \( (w_r, \alpha_r) \) to accommodate this decision, thereby shifting the priority from signaling to cost savings. It turns out that the vendor maximizes her payoff either in a pooling equilibrium with full warranty or in a payoff-equivalent separating equilibrium. Furthermore, any pooling contract comprising a partial warranty is not offered in equilibrium, because the reliable vendor always does better by deviating to an alternate contract (which may itself not survive in equilibrium) with slightly higher warranty coupled with a slightly higher fixed fee, which the unreliable vendor finds too costly to mimic.

In summary, the vendor who adopts RBC is able to signal product reliability without affecting supply-chain efficiency only if the inventory is verifiable. If not, the vendor’s discretionary inventory choice exacerbates the problem of incentive misalignment that already exists with the signaling challenge, leading to an outcome where the vendor is not better off than in a pooling equilibrium with an underinvestment in inventory that causes economic inefficiency.

### 6.2. Performance-Based Contract

The players’ payoff functions under PBC remain the same as those of the verifiable inventory case. With inventory no longer included in the contract, however, the vendor’s discretionary choice of \( s_r \) will influence her choices for the remaining contract terms: the fixed fee \( w_r \), and the performance penalty rate \( v_r \).

Recall from §5.2 that when inventory is verifiable and is included in the contract, PBC allows the reliable vendor to successfully signal her type and achieve economic efficiency. A key reason why this happens is that the penalty rate \( v_H \) offered by the reliable vendor is used only as a signaling device and serves no other purpose. Once inventory is not verifiable, however, \( v_H \) assumes another role in addition to that of signaling: providing a financial incentive to hold inventory. Thus, when inventory is unverifiable, the penalty rate serves a dual role. On the one hand, the reliable vendor prefers to signal by setting a high penalty rate in order to deter mimicking by the unreliable type. On the other hand, her temptation to forgo investment in non-contractible inventory exerts a downward pressure on the penalty rate. Our next result reveals the equilibrium outcome that emerges from this trade-off.

**Proposition 5.** When inventory is unverifiable, the PBE of the signaling game under PBC that satisfies the conditions in §3.5 is unique, separating, and inefficient. In this equilibrium, \( \overline{s}_L = s^*_L, \overline{s}_H > s^*_H, w^*_L < w^*_H, \) and \( \chi = v^*_L < v^*_H. \) As a result, it is not possible for the reliable vendor to recover her first-best payoff: \( V^*_H < V_H. \) The buyer’s payoff is equal to his outside option \( \theta. \)
As the proposition describes, the resulting equilibrium is separating but fails to achieve efficiency; the reliable vendor destroys economic value in order to signal her type. Moreover, the reliable vendor overinvests in inventory in equilibrium \( s^*_H > s_H \). This is in contrast to the verifiable inventory case, in which we found that the reliable vendor is able to achieve both separation and efficiency with the first-best inventory under PBC. Therefore, when inventory cannot be contracted upon, the two competing forces exerted upon the penalty rate \( v_H \) do not prevent the reliable vendor from signaling her type: she eventually does so at the expense of the increased cost of maintaining inventory. Note also that overinvestment in inventory is exactly the opposite outcome of the equilibrium under RBC, where it was found that underinvestment arises.

To understand why overinvestment arises in equilibrium, we revisit the observation that PBC supports signaling through the degree of outage risk shared by the vendor. Note that separation cannot be achieved by agreeing to a penalty that induces first-best inventory investment (i.e., \( v^*_H = \chi \)), since the first-best outcome described in Proposition 1 is not incentive compatible. Similarly, any attempt to separate with a lower penalty will also not be incentive compatible – the unreliable vendor will find it even easier to mimic. Therefore, the only option remaining for the reliable vendor is to increase the penalty rate sufficiently to deter mimicking by the unreliable vendor.

A direct consequence of this overinvestment in inventory is that the payoff appropriated by the reliable vendor is less than the first-best level. However, this is not the only factor at play. Another reason for this gap in payoff stems from the renegotiation-proof requirement that forces the reliable vendor to surrender rents to the unreliable vendor. Although this characteristic is typical of pooling equilibria in which the unreliable vendor benefits from the fact that the buyer cannot identify her type, it also manifests itself in renegotiation-proof separating equilibria, as a mechanism to ensure that no other Pareto-improving proposal exists after the contract is signed. This point is numerically illustrated in Figures 1 and 2 using the parameter values in Table 2, which are typical in the civil aerospace industry. In this example, the inefficiency associated with overinvestment in inventory is substantial (see Figure 1), but smaller than the overall loss of rents that asymmetric information inflicts on the reliable vendor (see Figure 2). The additional loss is due to the rents surrendered to the unreliable vendor to ensure renegotiation-proofness. We refer interested readers to Beaudry and Poitevin (1993) for further details.

Overall, we find that the interaction between the vendor’s discretionary inventory choice and her signaling incentive creates subtle dynamics that impact the relative efficiency of after-sales service contracts in nontrivial ways. While system efficiency cannot be attained under either RBC or PBC, they should be distinguished in that the equilibrium outcomes under the two contract types are qualitatively different: whereas RBC leads to underinvestment in inventory, the opposite is true under PBC. Moreover, the buyer under RBC may find himself unable to obtain information about
In closing, we ask the following question: Given that neither RBC nor PBC gives the reliable vendor a clear advantage when inventory cannot be included in a contract, which contract type will she prefer? The answer is given below.

Corollary 1. The reliable vendor’s equilibrium payoff under PBC with unverifiable inventory (Proposition 5) is greater than that under RBC with unverifiable inventory (Proposition 4).

Thus, the reliable vendor finds that the ability to signal her type with PBC more than compensates for the cost associated with overinvestment in inventory and any benefit presented by RBC, which does not improve upon a pooling outcome. Note that, although no strict ordering of payoffs such as in Corollary 1 exists for the unreliable vendor, a similar conclusion is made under reasonable parameter value combinations (such as those in Table 2): the unreliable vendor’s payoff is greater under PBC than under RBC, except in extreme cases (e.g., $p \to 0$ and $\mu_L \gg \mu_H$).

<table>
<thead>
<tr>
<th>Definition</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi$ Backorder cost</td>
<td>$2,000,000$ per month</td>
<td>Estimated based on revenue per aircraft figures reported by airlinefinancials.com: an independent airline industry consulting firm.</td>
</tr>
<tr>
<td>$c$ Cost per spare</td>
<td>$70,000$ per month</td>
<td>Estimated based on engine lease-rental rates (Canaday 2010) and cost of a spare engine (Adler et al. 2009).</td>
</tr>
<tr>
<td>$1/\mu_H$ Expected time between failures for $H$-type vendor</td>
<td>0.5–5 years</td>
<td>Within range reported in Guajardo et al. (2012).</td>
</tr>
<tr>
<td>$1/\mu_L$ Expected time between failures for $L$-type vendor</td>
<td>0.5 years</td>
<td>Within range reported in Guajardo et al. (2012).</td>
</tr>
<tr>
<td>$r$ Inconvenience cost</td>
<td>$175,000$</td>
<td>Engine replacement time $\sim 2$ days; linear extrapolation at rate of $$61$ per minute of delay (Ramdas et al. 2013).</td>
</tr>
<tr>
<td>$N$ Number of engines</td>
<td>150</td>
<td>Order size for large airline (Kim et al. 2007).</td>
</tr>
<tr>
<td>$M$ Cost per unscheduled maintenance</td>
<td>$800,000$</td>
<td>Estimated based on industry expert input and consistent with figures reported in Hopper (1998).</td>
</tr>
<tr>
<td>$t$ Average repair lead time for engine overhaul</td>
<td>3 months</td>
<td>Based on numbers in Adamides et al. (2004) (accounting for operational improvements).</td>
</tr>
<tr>
<td>$F_r(x)$ Failure distributions</td>
<td>-</td>
<td>Normal approximation to Poisson distribution.</td>
</tr>
<tr>
<td>$p$ Buyer’s prior belief that the vendor is $L$-type</td>
<td>50%</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 Parameter Values

![Figure 1](image1.png) Increase in reliable vendor’s inventory cost under PBC when inventory is unverifiable compared to first-best. Parameter values shown in Table 2.

![Figure 2](image2.png) Vendor’s payoffs under PBC when inventory is unverifiable. Buyer’s revenues are normalized to zero. Parameter values shown in Table 2.

product reliability (i.e., pooling equilibria cannot be ruled out) but the buyer under PBC receives an unambiguous signal about reliability.

Figure 1 Increase in reliable vendor’s inventory cost under PBC when inventory is unverifiable compared to first-best. Parameter values shown in Table 2.

Figure 2 Vendor’s payoffs under PBC when inventory is unverifiable. Buyer’s revenues are normalized to zero. Parameter values shown in Table 2.
7. Extensions: Efficiency Enhancement Strategies

Thus far, our focus has been on the properties of the equilibrium outcomes that arise when the contract type (RBC or PBC) and verifiability of inventory have been exogenously specified. We found that while system efficiency is ensured when inventory is verifiable, neither PBC nor RBC leads to efficiency when inventory is unverifiable. Therefore, a natural course of action for reducing inefficiency is to invest in verifiability of inventory. We investigate this option in the context of either the vendor or the buyer making a choice of contract type, in order to highlight how inventory verifiability shapes their preferences. In case the vendor manages inventories across multiple buyers, an alternative option for reducing inefficiency is to utilize inventory pooling. In this section we investigate these two efficiency enhancement strategies and discuss their implications.

7.1. Endogenizing Inventory Verifiability

We study an extended game in which either the vendor or the buyer is empowered to decide whether inventory should be made verifiable and select the contract type before actual contract terms are determined. Given that the vendor is the principal in our setting, it is natural to consider the case in which she makes these choices. However, motivated by the practices in the civil aerospace sector, in addition to the vendor’s choice we also consider the case in which the buyer wields some bargaining power (e.g., a large airline company) by deciding the contract type and inventory verifiability.\(^6\)

A key addition in this setting is that inventory verifiability does not come for free. An irreversible fixed cost \(K\) is incurred by the vendor to make the inventory decision observable by the buyer and verifiable by a third party. In practice, verifiability is often achieved by purchasing and implementing information systems; e.g., CD Aviation Services offers the SilverSky program which enables customers to track the progress of their repair job with detailed status reports and pictures via secure Internet access anytime and from anywhere in the world (Adler et al. 2009, p. 18). Alternatively, inventory can be made verifiable by maintaining it on the customer’s site (Adams 2008, Canaday 2010). We refer to \(K\) as the “cost of investing in verifiability.” To focus on nontrivial cases, we make a mild assumption that the fixed cost is lower than the system-wide surplus generated, irrespective of the vendor type (i.e., \(K < V_L\)).

Endogenizing inventory verifiability along with contract choice alters the sequence of events in the game. Note that, in principle, such a choice itself could reveal information and it may therefore generate equilibrium outcomes that are different from those we obtained in the previous sections. The altered sequence of events is as follows.

1. Nature reveals to the vendor her reliability (type): \(\tau \in \{L, H\}\).

\(^6\) Our modeling assumption is similar to that in Aksin et al. (2008), who study a call center setting in which the contractor (principal) first offers the service provider (agent) a choice between two contract types, and then, depending upon the contract type chosen by the agent, the principal specifies the contract parameters.
2. Player $i \in \{\text{vendor, buyer}\}$ announces whether or not inventory should be made verifiable and chooses a contract type (RBC or PBC), but not the specific contract terms. Player $i$'s identity is exogenously specified.

3. The vendor incurs an irreversible fixed cost $K$ if verifiable inventory is chosen; if player $i$ is the vendor, then the buyer updates his beliefs about the vendor's type.

4. The vendor offers the buyer specific contract terms (possibly type-contingent).

5. Based on the proposed contract terms, the buyer updates his beliefs about the vendor's reliability (type), and accepts or rejects the contract after a possibly infinite round of renegotiations.

6. Products are deployed, failures occur, and repair and maintenance takes place; transfer payment is made by the buyer and final payoffs are realized by both players.

The analysis of this extended game builds on the results obtained in the previous two sections. The key result is summarized below.

**Proposition 6.** In the game featuring choices on inventory verifiability and contract type:

- a) Suppose that it is the vendor who makes the choices. A threshold value $K_v > 0$ exists, such that if $K > K_v$, both vendor types select PBC and choose not to invest in verifiability. The resulting equilibrium is separating as described in Proposition 5. If $K < K_v$, on the other hand, the reliable vendor signals her type by choosing to invest in verifiability coupled with either RBC or PBC, whereas the unreliable vendor signals by choosing not to invest in verifiability in conjunction with PBC. In any of these cases, the buyer's payoff is equal to his outside option $\theta$.

- b) Suppose that it is the buyer who makes the choices. The buyer then opts for RBC with verifiable inventory, and the payoffs are determined as per the separating equilibrium in Proposition 2, except that the vendor's payoff is reduced by $K$.

The proposition shows that, despite the efficiency gain that inventory verifiability brings, it may create diverging preferences between the vendor and the buyer about contract types: the vendor prefers PBC whereas the buyer prefers RBC. From the vendor's perspective, PBC dominates RBC because the former allows the reliable vendor to signal her type more efficiently. Moreover, endowing the vendor with the ability to choose inventory verifiability and the contract type gives her an extra lever, besides contract parameters, with which she can signal her reliability. This is because the verifiability investment acts as a credible signal of reliability when the cost of verifiability $K$ is relatively low compared to the value lost due to investory overinvestment, which we identified in §6. In contrast to the vendor who prefers PBC, the buyer prefers RBC because the combination of RBC with verifiable inventory allows the buyer to retain positive rents in equilibrium, whereas PBC does not, as we found in §5.

Our predictions are consistent with what is observed in practice. As has been reported by GAO (2004) and others, when product technology is new and information on product reliability is not
readily available, buyer organizations tend to opt for RBC over PBC. Moreover, the above finding also matches with the growing trend in the aerospace industry where many vendor organizations (e.g., Rolls-Royce, International Aero Engines, etc.) are moving towards offering PBC exclusively (Adler et al. 2009). Although there are factors other than signaling that may explain these observations, our analysis provides theoretical support for these industry practices when reliability signaling is a significant contributor to the firms’ decisions.

7.2. Pooling Inventory Across Multiple Buyers

When inventory verifiability investment is prohibitively expensive but the vendor transacts with multiple buyers, she may consider adopting the inventory pooling strategy. Indeed, this practice is widely used among vendors of products with mature technology. While pooling is relevant for both RBC and PBC in principle, in our model context its impact under RBC is limited because, as demonstrated in §6.1, the vendor has a minimal incentive to maintain a high level of inventory under RBC. On the other hand, pooling effect may be magnified under PBC because it encourages inventory overinvestment. For this reason, we restrict our attention to PBC in the following discussion.\footnote{In addition, we do not study the pooling strategy when inventory is verifiable because the typical means for achieving verifiability, i.e., maintaining inventory on the customer’s site, makes it impractical to pool inventory across buyers. Besides, when reliability is not common knowledge and inventory is contractible, pooling would require intricate considerations around multi-party contracting and contentious inventory sharing rules. The above factors effectively make pooling infeasible, as is reflected in the aerospace industry where, for engines with new technology, buyers opt for tangible on-site assets (dedicated spares), as opposed to participating in a pool (Canaday 2010).}

We modify our model to illustrate the key trade-offs associated with inventory pooling by making two simplifying assumptions that enable analytical tractability. First, we assume that the vendor trades simultaneously with \(k\) identical buyers, each purchasing \(N\) product units. Hence, the total sales for the vendor are equal to \(Q = kN\) units. The number \(k\) is given and is common knowledge. The sequence of events and equilibrium identification strategy remain the same as in §3.5. Since all buyers are identical, we focus on symmetric equilibria in which all buyers receive exactly the same (potentially type-dependant) contract terms. Second, to quantify the benefits of pooling, we focus on a scenario in which product failures for each buyer occur as a Poisson process with the common failure rate \(\mu\). To facilitate the game-theoretic analysis of our paper we approximate the Poisson distribution with the Normal distribution.

As contract offers are made to each of the \(k\) buyers simultaneously and independently, the dynamics of the signaling game are unaffected by the presence of multiple buyers. The only departure from the analysis of the previous sections is a possible cost saving through inventory pooling and its impact on signaling effectiveness. The question naturally arises: Is pooling always beneficial from the vendor’s perspective? We address this question with our next result.
Figure 3  The Value of Pooling: difference in the reliable vendor’s payoffs (monthly) when pooling inventory across $k \in \{1, 2, 3, 4\}$ buyers. Parameters shown in Table 1. Left-hand (Right-hand) panel represents the expensive (inexpensive) backorders case with $\chi = $2,000,000 ($\chi = $72,500).

**Proposition 7.** When trading with multiple identical buyers, the reliable vendor is better off by pooling inventory if and only if

$$\nabla_H - V^*_H > \Delta, \tag{3}$$

where $V^*_H$ is the equilibrium payoff for the reliable vendor facing a single buyer (specified in Proposition 5), $\nabla_H$ is the corresponding first-best payoff (specified in Proposition 1), and $\Delta = p(\mu_L - \mu_H)r + 2c\mu_H l - \chi E_{H}(B|\bar{x}_H)$.

The proposition shows that when inventory is unverifiable, inventory pooling is beneficial for the reliable vendor if and only if the efficiency gap from contracting with an individual buyer is greater than a threshold amount; otherwise, the reliable vendor ends up worse off by pooling inventory across multiple buyers. To understand the intuition behind this result, we need to examine the two competing effects inventory pooling has on payoffs. The first effect, which benefits both the reliable vendor and the unreliable vendor, comes from the fact that a larger pool of products brings more predictability of unplanned failure events. The cost saving that follows is inherently lower for the reliable vendor than for the unreliable vendor because the variability that the former faces is smaller to begin with. The second effect, which benefits the unreliable vendor at the expense of the reliable vendor, is due to the change in signaling costs caused by inventory pooling. Note that the unreliable vendor will find it relatively cheaper to pretend she is reliable if she pools inventory across multiple buyers: the larger pool lowers her cost of matching the high penalty rate that the reliable vendor proposes. This in turn makes it more difficult for the reliable vendor to enjoy the benefit of pooling, because lowering the penalty rate to take advantage of such a benefit exposes her to possible mimicking by the unreliable vendor. Thus, pooling will actually increase the cost of signaling for the reliable type. Which of these two effects of pooling – uncertainty reduction and more costly signaling – prevails depends on the size of the efficiency gap as specified in (3).

The value of pooling is demonstrated in Figure 3. Pooling is beneficial when inventory is relatively inexpensive compared to the cost of backorders (left-hand panel in Figure 3), i.e., $c/\chi$ and
hence $\Delta$ in (3) is small. (Furthermore, as also evident from Figure 3, pooling exhibits the usual diminishing returns to scale with respect to the number of buyers, $k$.) Nevertheless, as the problem of asymmetric information becomes more pronounced, this benefit declines and can even become negative; e.g., when inventory is relatively expensive compared to the cost of backorders (right-hand panel in Figure 3). Hence, we conclude that while inventory pooling is generally beneficial to the vendor, there are situations in which pooling backfires. This observation serves as a reminder that the unique challenges arising from asymmetric information about new product reliability require actions that may deviate from the usual efficiency enhancement strategies.

8. Conclusions
In this paper we study how two widely-used after-sales service contracts, resource-based contracts (RBC) and performance-based contracts (PBC), can be used to signal reliability of a newly developed product. Our analysis uncovers new dynamics that arise from the interaction between the vendor’s desire to signal reliability using contract terms, and her incentive to minimize costs (equivalently maximize profit) through investment in spares inventory.

We find that RBC and PBC differ significantly in their ability to signal reliability, and the transparency of inventory plays a crucial role. When inventory is verifiable, and thus signaling private information about new product reliability is considered in isolation, both RBC and PBC can be used as an efficient signaling mechanism. However, one difference exists between the two contracts: it is possible for the vendor to fully extract rent from the buyer under PBC, whereas the same is not true under RBC. Thus, from the vendor’s perspective, PBC turns out to be a more effective signaling mechanism than RBC. The driver behind this outcome is the structural difference between the two contract types: RBC allows signaling via the vendor’s share of the repair risk (determined entirely by reliability), whereas PBC permits signaling through the vendor’s share of the outage risk (determined jointly by reliability and inventory choice). Despite the differences, both RBC and PBC give rise to efficient separating equilibria.

The property of efficient separating equilibria no longer holds when the vendors inventory choice is discretionary, because this private action interacts with her ability to signal private information about reliability. In this setting, PBC creates an incentive for the vendor to overinvest in spares inventory; a large penalty rate associated with prolonged product outage is offered to the buyer so as to assure him of high product reliability, but as a consequence the vendor needs ample amount of spares in order to increase the product up-time and reduce the impact of heavy penalties that may follow. This is in contrast to what happens with RBC, under which the vendor is provided with little incentive to carry spares inventory because the contract terms do not hold the vendor accountable for prolonged outages. The net outcome is that neither contract results in efficiency
when inventory is unverifiable. Whereas PBC puts a disproportionate emphasis on reliability signaling over inventory savings, RBC does the opposite. Nevertheless, we find that the vendor’s payoffs are in general higher under PBC rather than RBC, suggesting that in this case too PBC is a more effective signaling mechanism than RBC.

Our analysis suggests that the key to bring efficiency to the supply chain is inventory transparency; by making inventory verifiable, contracts can be structured such that incentive distortions disappear and effective signaling is enabled. However, this improvement option may create diverging preferences for contracts. Specifically, when inventory is made verifiable the vendor prefers PBC since it allows her to extract more rents from the buyer, while the buyer prefers RBC since it leaves him with a positive surplus. This finding is consistent with reports of aerospace industry practices which indicate that buyer organizations typically opt for RBC when they acquire newly developed products.

An alternative way to bring efficiency is to utilize inventory pooling, in case the vendor manages inventories across multiple buyers. Although the benefit of pooling is well-known, we demonstrate that this strategy can backfire in our setting: it may hinder reliability signaling and exacerbate inefficiency. This happens because the pooling effect benefits the unreliable vendor disproportionately, thus making it easier for her to misrepresent her product reliability. Therefore, inventory pooling should be adopted with caution when firms operate in an environment in which products are newly developed.

Our analysis adds a new dimension to the theory and practice of after-sales product support contracting. By shifting the focus from the management of mature products to the management of newly developed products, we identify a new set of challenges which have been overlooked in the existing literature. With the after-sales service business continually growing and new products being introduced at faster rates, we believe that the managerial insights generated from our analysis not only serve as useful guidelines for practitioners, but open up new possibilities for future work.

9. Appendix: Proofs

Before proceeding with the proofs, we find it useful to first prove the following Lemma.

**Lemma 1.** For any value of \( u \geq c \), when \( O_L \) and \( O_H \) satisfy the hazard rate ordering of (1), then \( \phi(u) := u[E_H(B|s_H(u)) - E_L(B|s_L(u))] + c(s_H(u) - s_L(u)) < 0 \), where, for \( \tau \in \{L, H\} \) \( s_\tau(u) = \arg \max_s [-uE_\tau(B|s) - cs] = F^{-1}_\tau(1 - \frac{c}{u}) \). Furthermore, when \( O_L \) and \( O_H \) also satisfy the EW order of (2), then \( \phi(u) \) is non-increasing.

**Proof of Lemma 1:** Since \( O_L \) and \( O_H \) follow a hazard rate ordering that implies first-order stochastic dominance (FOSD), for any \( u > 0 \) we therefore have \( s_H(u) \leq s_L(u) \). Furthermore, \( \phi(u) = u[E_H(B|s_H(u)) - E_L(B|s_L(u))] + c(s_H(u) - s_L(u)) = u \int_{s_H(u)}^{s_L(u)} (1 - F_H(x)) dx + \)
the definitions of \( s_u \) or \( s_l \) interval \((−u, u)\) with respect to inventory is unique as the objective is concave in \( u \). Therefore, the optimal inventory to keep is equal to the first-best level, i.e., \( L_s \). Taking the derivative of \( φ(u) \) with respect to \( u \) gives \( φ'(u) = \frac{\partial}{\partial u} \left[ E_H(B|s_H(u)) - E_L(B|s_L(u)) \right] \leq 0 \).

**Proof of Proposition 1**: The objective function is increasing in \( w \), therefore, the (IR) constraint must be binding: \( w_r = (\chi E_r(B|s_r) + µ_r) \). Using this and maximizing the objective with respect to inventory \( s_r \), we obtain \( s_r = F_r^{-1}(1 - \frac{ε}{\chi}) \) when \( χ \geq c \) and zero otherwise. The solution is unique as the objective is concave in \( s_r \). From (1), which implies FOSD, we conclude that \( s_L > s_H \). The vendor’s payoff is \( V_r = -θ - \chi E_r(B|s_r) - c s_r - µ_r (r + M) \). Furthermore, \( V_L - V_H = \chi [E_H(B|s_H) - E_L(B|s_L)] + (µ_H - µ_L) (r + M) = φ(χ) + (µ_H - µ_L) (r + M) \), using Lemma 1 for \( u = χ \) and \( µ_H < µ_L \), we conclude that \( V_L - V_H < 0 \).

**Proof of Proposition 2**: In this proof we adopt the classic “one-shot” signaling treatment (without Step 4 in §3.5) to determine the equilibrium outcome, as opposed to the signaling-with-renegotiation framework proposed by Beaudry and Poitevin (1993). This is because, as per the sequence of events in §3.5, the accepted proposal in Step 3 turns out to be efficient and, therefore, renegotiation-proof. In a separating equilibrium, if it exists, the vendor is able to credibly signal her type. Given that the L-type vendor has credibly communicated her type to the buyer, the best that she can do is to extract all the surplus by solving the following problem:

\[
\max_{w_L, s_L, α_L \geq 0} [w_L - (1 - α_L)µ_L M - cs_L], \text{subject to (IR}_L): -w_L - α_L µ_L M - \chi E_L(B|s_L) - µ_L r \geq \theta.
\]

The constraint will be binding at optimum (otherwise the vendor can increase the fixed fee \( w_L \) and improve her payoff), therefore \( w_L^* = -α_L µ_L M - \chi E_L(B|s_L) - µ_L r - \theta \), and the objective becomes \( \max_{s_L, α_L \geq 0} [-\chi E_L(B|s_L) - µ_L (r + M) - θ - cs_L] \). This function does not depend on \( α_L \) and is concave in \( s_L \). Therefore, the optimal inventory to keep is equal to the first-best level, i.e., \( s_L^* = s_L \).

The unreliable vendor makes her first-best payoff, \( V_L \).

Now consider the problem of the H-type vendor. If there exist no consistent separating deviations as per the “intuitive criterion,” then the H-type’s contract must solve the following problem:

\[
\max_{w_H, s_H, α_H \geq 0} [w_H - (1 - α_H)µ_H M - cs_H], \text{subject to (IR}_H): -w_H - α_H µ_H M - \chi E_H(B|s_H) - µ_H r \geq \theta, \text{(IC}_L): \frac{\partial}{\partial w_H} \geq w_H - (1 - α_H)µ_L M - cs_H.\]

Furthermore, for a separating equilibrium the H-type’s contract needs to be incentive-compatible, (IC\(_H\)): \( w_H + α_H µ_H M - cs_H \geq α_L (µ_H - µ_L) M - \chi E_L(B|s_L) - µ_L r - cs_L - \theta \). Since the objective function is increasing in \( w_H \) and an increase in \( w_H \)
cannot violate (IC$_H$), we must conclude that either (IR$_H$) or (IC$_L$) is binding. We consider these possibilities in turn.

Case A: (IC$_L$) is binding. This implies $w_H = -\chi E_L(B|\bar{s}_L) - \mu_L r - \theta - c(\bar{s}_L - s_H) - \alpha_H \mu_L M$ and the optimization problem becomes $\max_{s_H, \alpha_H \geq 0} \{-\chi E_L(B|\bar{s}_L) - \mu_L r - \theta - c\bar{s}_L - \mu_H M - \alpha_H (\mu_L - \mu_H) M\}$, subject to (IR$_H$): $\chi [E_L(B|\bar{s}_L) - E_H(B|s_H)] + c(\bar{s}_L - s_H) + (\mu_L - \mu_H)(\alpha_H M + r) \geq 0$, and (IC$_H$): $(\alpha_L - \alpha_H)(\mu_L - \mu_H) M \geq 0$. Note that (IC$_H$) implies that any feasible solution requires $0 \leq \alpha_H \leq \alpha_L$. Furthermore, the objective function is independent of $s_H$ and is decreasing in $\alpha_H$. Starting with any feasible solution such that $\alpha_H > 0$, decreasing $\alpha_H$ cannot violate (IC$_H$), and decreasing $\alpha_H$ does not violate (IR$_H$) as long as an appropriate inventory, $s_H$, is set. Therefore $\alpha_H = 0$ at optimum, provided the following condition is satisfied $\chi [E_L(B|\bar{s}_L) - E_H(B|s_H)] + c(\bar{s}_L - s_H) + (\mu_L - \mu_H) r \geq 0$. This condition implies that $s_H$ should satisfy $\max\{0, s^m\} \leq s_H \leq s^M$, where $s^m$ and $s^M$ are the two roots of the equation $cs + \chi E_H(B|\bar{s}) = (\mu_L - \mu_H) r + \chi E_L(B|\bar{s}_L) + c\bar{s}_L$. It is easy to verify that $\max\{0, s^m\} \leq \bar{s}_H \leq s^M$ by setting $u = \chi$ in Lemma 1, hence $\alpha^*_H = 0$. Note that $s^m$ could be negative depending on the model parameters. The payoff of the reliable vendor is given by $V_H = -\chi E_L(B|\bar{s}_L) - c\bar{s}_L - \mu_L r - \mu_H M - \theta = V_L + (\mu_L - \mu_H) M < V_H$.

Case B: (IR$_H$) is binding. This case does not contain any further solutions. Out of all the PBE identified above, only the efficient ones (i.e., those with $s^*_H = \bar{s}_H$) are renegotiation-proof. We therefore eliminate all others. □

**Proof of Proposition 3:** In this proof we adopt the classic “one-shot” signaling treatment (without Step 4 in §3.5) to determine the equilibrium outcome, as opposed to the signaling-with-renegotiation framework proposed by Beaudry and Poitevin (1993). This is because, as per the sequence of events in §3.5, the accepted proposal in Step 3 turns out to be efficient, and therefore renegotiation-proof. We consider the possibility of recovering first-best rents in a separating equilibrium: $(w^*_H, v^*_H, s^*_H)$ and $(w^*_L, v^*_L, s^*_L)$. Since inventory is observable and verifiable it is straightforward to contract on its first-best levels: $\bar{s}_H$ and $\bar{s}_L$. However, the transfer payments need to adhere to the incentive compatibility (IC$_\tau$: $V_\tau \geq w_\tau + v_{\tau'} E_r(B|s_{\tau'}) - \mu_\tau M - cs_{\tau'}$) and participation constraints (IR$_\tau$: $-w_\tau + (v_\tau - \chi) E_r(B|s_\tau) - \mu_\tau r \geq \theta_\tau$) for $\tau, \tau' \in \{L, H\}, \tau \neq \tau'$.

Consider the following contract parameters: $w^*_H = -\theta - \mu_H r + (v^*_H - \chi) E_H(B|\bar{s}_H)$, $w^*_L = -\theta - \mu_L r + (v^*_L - \chi) E_L(B|\bar{s}_L)$. For these choices of fixed fee, the participation constraints are binding, and this implies that both types of vendor recover their first-best outcomes, provided the incentive compatibility (IC) constraints are satisfied. We now check (IC$_H$). Plugging in the appropriate values, we obtain $v^*_H [E_L(B|\bar{s}_L) - E_H(B|\bar{s}_L)] \leq \chi [E_L(B|\bar{s}_L) - E_H(B|\bar{s}_H)] + (\mu_L - \mu_H) r + c(\bar{s}_L - \bar{s}_H)$. Similarly, (IC$_L$) implies $v^*_L [E_H(B|\bar{s}_H) - E_L(B|\bar{s}_L)] \geq \chi [E_L(B|\bar{s}_L) - E_H(B|\bar{s}_H)] + (\mu_L - \mu_H) r + c(\bar{s}_L - \bar{s}_H)$. The conditions above are satisfied for any $v^*_L$ and $v^*_H$ such that: $v^*_L [E_L(B|\bar{s}_L) - E_H(B|\bar{s}_L)] \leq \chi [E_L(B|\bar{s}_L) - E_H(B|\bar{s}_H)] + (\mu_L - \mu_H) r + c(\bar{s}_L - \bar{s}_H)$ ≤
Proof of Proposition 4: As per the framework for signaling-with-renegotiation (Beaudry and Poitevin 1993), when the one-shot (without Step 4 in §3.5) equilibrium is not efficient then it may not be renegotiation-proof. Since this is the case (proof-omitted for brevity), we follow Beaudry and Poitevin (1993) to identify the renegotiation-proof outcome. Note that for $\tau \in \{L, H\}$ the vendor’s payoff is $V_\tau = w + \alpha \mu_\tau M - c s_\tau - \mu_\tau M$, which is decreasing in inventory $s$. Therefore the optimal inventory is $s^*_H = s^*_L = 0$. The payoff of the buyer is: $U_\tau = -w_\tau - (\alpha_\tau M + \chi l + r)\mu_\tau$, where we have used $E_\tau(B|0) = \mu_\tau l$, where $l$ is the expected repair lead time using Wald’s equation and the fact that each one of the $\mu_\tau$ expected failures will generate an expected period $l$ of downtime.

Then, $\frac{\partial U_\tau}{\partial w} = -1$; $\frac{\partial U_\tau}{\partial \alpha} = -\mu_\tau M$; $\frac{\partial U_\tau}{\partial s} = 1$; and $\frac{\partial U_\tau}{\partial \mu} = \mu_\tau M$. It is straightforward to verify that Assumption 3A and Assumption 3B (Case RS) in Beaudry and Poitevin (1993) (essentially single-crossing property for the vendor’s payoff) are satisfied. We now solve for the contract parameters, following the scheme laid out in Proposition 3 of Beaudry and Poitevin (1993). Also, we define $F_p(.) = p F_L(.) + (1-p) F_H(.)$ with corresponding interpretation for $\mu_p$ and $E_p(B|0) = p E_L(B|0) + (1-p) E_H(B|0)$. We first solve for the reliable vendor’s contract parameters as follows: $\max_{w, \alpha \geq 0} [w + \alpha \mu_H M - \mu_H M]$, subject to (IR): $-w - \alpha \mu_p M - \chi \mu_p l - \mu_p r \geq \theta$. Clearly (IR) is binding at optimality, otherwise an increase in $w$ will improve the objective. Therefore $w^*_H = -\theta - \alpha \mu_p M - \chi \mu_p l - \mu_p r$. Plugging this back into the objective, we get: $\max_\alpha [\text{constant} + \alpha (\mu_H - \mu_p) M]$. Therefore, $\alpha^*_H = 0$ and $w^*_H = -\theta - \chi \mu_p l - \mu_p r$.

Next we characterize the unreliable vendor’s contract parameters by solving the following optimization problem: $\max_{w, \alpha \geq 0} [w + \alpha \mu_1 M - \mu_1 M]$, subject to: $-w - \alpha \mu_1 M - \chi \mu_1 l - \mu_1 r \geq -w^*_H - \chi \mu_1 l - \mu_1 r$; and $-w - \alpha \mu_p M - \chi \mu_p l - \mu_p r \geq \theta$. We ignore the second constraint and verify that is satisfied at optimality. As before, the first constraint is clearly binding at optimality, such that $w^*_L + \alpha^*_H \mu_1 M = w^*_H$. This determines the value of the objective function entirely. Further, the second constraint is satisfied for $\alpha^*_L \geq 0$. Hence, $\alpha^*_L = 0$ will give rise to a pooling equilibrium, but there exist a continuum of separating equilibria with $\alpha^*_L > 0$ that are payoff-equivalent to the pooling equilibrium. □

Proof of Proposition 5: We define $F_p(.) = p F_L(.) + (1-p) F_H(.)$, $\mu_p = p \mu_L + (1-p) \mu_H$, and $E_p(B|s(v)) = p E_L(B|s_L(v)) + (1-p) E_H(B|s_H(v))$. As per the framework for signaling-with-renegotiation (Beaudry and Poitevin 1993), when the one-shot (without Step 4 in §3.5) equilibrium is not efficient then it may not be renegotiation-proof. Since this is the case (proof-omitted for brevity), we follow Beaudry and Poitevin (1993) to identify the renegotiation-proof
outcome. Recall that for \( \tau \in \{L, H\} \), \( V_\tau = w - vE_\tau(B|s_\tau(v)) - cs_\tau(v) - \mu_\tau M \); and \( U_\tau = -w + (v - \chi)E_\tau(B|s_\tau(v)) - \mu_\tau r \), where inventory is \( s_\tau(v) = F_{\tau}^{-1}(1 - \frac{1}{v}) \). Then, \( \frac{\partial g}{\partial w} = -1, \frac{\partial p}{\partial v} = E_\tau(B|s_\tau(v)) - (v - \chi)\frac{1}{v^2} \frac{F_{\tau}(s_\tau(v))}{f_{\tau}(s_\tau(v))}, \frac{\partial v}{\partial w} = 1, \) and \( \frac{\partial v}{\partial w} = -E_\tau(B|s_\tau(v)) \). Under condition (2) (i.e., \( E_L(B|s_L) > E_H(B|s_H) \)), it is straightforward to verify that Assumption 3B (Case RS) in Beaudry and Poitevin (1993) (essentially single-crossing property for the vendor’s payoff) is satisfied. Using the hazard rate order and increasing hazard rate property, Assumption 3A in Beaudry and Poitevin (1993) is also satisfied, provided \( v \geq \chi \). We verify that \( v < \chi \) is never an equilibrium outcome, hence we can explicitly restrict the parameter space to \( v \geq \chi \). Consequently, the conditions for Proposition 3 in Beaudry and Poitevin (1993) are met, and we can conclude that the renegotiation-proof contracts are unique with respect to payoff.

We now solve for the contract parameters, following the scheme laid out in Proposition 3 of Beaudry and Poitevin (1993). We first solve for the reliable vendor’s contract parameters as follows:

\[
\max_{w,v} [w - vE_H(B|s_H(v)) - cs_H(v) - \mu_H M], \text{ subject to (IR): } -w + (v - \chi)E_p(B|s(v)) - \mu_p r \geq \theta.
\]

Clearly (IR) is binding at optimality, otherwise an increase in \( w \) will improve the objective. Therefore \( w_H^* = (v - \chi)E_p(B|s(v)) - \mu_p r - \theta \). Plugging this back into the objective function and taking first-order conditions, we obtain:

\[
\frac{(v - \chi)c}{v^2} \left( (1 - p) \frac{1 - F_H(s_H(v))}{f_H(s_H(v))} + p \frac{1 - F_L(s_L(v))}{f_L(s_L(v))} \right) = (E_L(B|s_L(v)) - E_H(B|s_H(v))). \tag{4}
\]

Since the RHS is always positive for finite \( v \), for the equation to have a finite solution it must be the case that \( v_H > \chi \), thereby resulting in inefficiency.

Next we characterize the unreliable vendor’s contract parameters by solving the following optimization problem:

\[
\max_{w,v} [w - vE_L(B|s_L(v)) - cs_L(v) - \mu_L M], \text{ subject to } -w + (v - \chi)E_L(B|s_L(v)) - \mu_L r \geq -w_H + (v_H - \chi)E_L(B|s_L(v_H)) - \mu_L r \text{ and } -w + (v - \chi)E_p(B|s(v)) - \mu_p r \geq \theta.
\]

We ignore the second constraint and verify that it is satisfied at optimality. As before, the first constraint is clearly binding at optimality, which can be written as \( w_H^* = (v - \chi)E_L(B|s_L(v)) + (1 - p)(v_H^* - \chi)E_H(B|s_H(v_H^*)) - E_L(B|s_L(v_H^*)) - \mu_p r - \theta \). Plugging this back into the objective function, we are left with the residual optimization problem \( \min_v [cs_L(v) + \chi E_L(B|s_L(v))] \), which has a unique minimum at \( v_L^* = \chi \). Finally, using (2) we have \( E_L(B|s_L(\chi)) > E_H(B|s_H(\chi)) \), which implies that the other second constraint, \(-w_H^* + (v_L^* - \chi)E_p(B|s(v_H^*)) - \mu_p r - \theta \), is satisfied.

**Proof of Corollary 1:** Let \( g = (v_H^* - \chi)E_p(B|s_H(v_H^*)) + \chi \mu_p l - v_H^*E_H(B|s_H(v_H^*)) - cs_H(v_H^*) \) denote the difference in payoffs for the \( H \)-type between the PBC contract of Proposition 5 and the RBC contract of Proposition 4. We will show that this is always non-negative. First note that \( v_H^*E_H(B|s_H(v_H^*)) + cs_H(v_H^*) = \min_s [v_H^*E_H(B|s) + cs] \leq v_H^*E_H(B|0) + c = v_H^*\mu_H l \). Therefore \( g = (v_H^* - \chi)E_p(B|s_H(v_H^*)) + \chi \mu_p l - v_H^*E_H(B|s_H(v_H^*)) - cs_H(v_H^*) \geq \) 0. Furthermore, from (4), note that \( \frac{\partial g}{\partial \mu_p} = 0 \). Therefore \( \frac{\partial g}{\partial p} = \frac{\partial g}{\partial v_H^*} \) is 0. Therefore \( \frac{\partial g}{\partial p} = \frac{\partial g}{\partial v_H^*} = \frac{\partial g}{\partial v_H^*} = \frac{\partial g}{\partial v_H^*} = (v_H^* -
Proof of Proposition 6: For part a) the choice in Step 2 is dictated by the preferences of the two types. For the \(H\)-type, RBC is dominated by PBC. In the case of verifiable inventory this due to the fact that the \(H\)-type can extract all rents with PBC but not with RBC, coupled with the fact that both RBC and PBC generate the same total value (i.e., they are both efficient). In the case of unverifiable inventory, this follows from Corollary 1. Thus, the \(H\)-type will never choose RBC over PBC. Comparing PBC verifiable with PBC unverifiable, the former is preferable for the \(H\)-type if
\[
\chi \mathbb{E}_H(B|s_H(v_H^*)) - \mathbb{E}_H(B|s_H(v_H^*)) + \chi(\mu_p - \mu_H)|l \geq 0.
\]
Since \(g \geq 0\) at \(p = 0\) and increasing in \(p\) we can therefore conclude that \(g \geq 0\) for all \(0 \leq p \leq 1\). □

Proof of Proposition 7: For part b) the buyer will choose RBC with verifiable inventory and thereafter the RBC separating equilibrium (Proposition 2) is played out. The buyer cannot do better in any other outcome. Note that the buyer’s choice is Step 2 reveals no information since he is the uninformed party. □

Before we proceed with the proof of Proposition 7 we find it useful to state and prove the following lemma.

**Lemma 2.** Under the assumptions of §7.2 (i.e., that the failures follow a Poisson distribution which we approximate with the Normal distribution), the performance-based penalty \(v_H^*\) given in Proposition 5 is independent of the number of products \(Q\) acquired by the buyer.
Proof of Lemma 2: We approximate the Poisson distribution with the Normal distribution, with following the density function for on-order inventory \( f_r(x; \mu_r) = \frac{1}{\sqrt{2\pi \mu_r t}} \exp \left( -\frac{(x-\mu_r t)^2}{2\mu_r t} \right) \) for \( \tau \in \{L, H\} \), where \( l \) is the expected repair lead time. The cumulative distribution function is \( F_r(x; \mu_r) \).

With \( k \) buyers, these functions are modified to \( f_r(x; k\mu_r) \) and \( F_r(x; k\mu_r) \). From the definition of \( \nu_H^* \) of \( (4) \) and recalling that \( F_r(s_r(v)) = 1 - c/v \) for \( \tau \in \{L, H\} \), we have:

\[
\tilde{\phi}(v, Q) = \frac{c^2(v - \chi)}{v^3} \left[ 1 - p \left( \frac{1}{f_H(s(v))} + \frac{p}{f_L(s(v))} \right) \right] - p \left[ \int_{s_L}^{\infty} (1 - F_L(x))dx + \int_{s_H}^{\infty} (1 - F_H(x))dx \right] = 0.
\]

If \( T_r \) is the expected time to failure for an engine of type \( \tau \), then for \( Q \) engines, the aggregate Poisson failure rate \( \mu_r = Q/T_r \). Also, making the observation that \( d\mu/dQ = 0 \), along with the Implicit Function Theorem, at \( \tilde{\phi} = 0 \) we obtain \( \frac{\partial \tilde{\phi}}{\partial v} dv + \frac{\partial \tilde{\phi}}{\partial \mu} d\mu = 0 \). We can conclude that \( dv/dQ = 0 \) if and only if: \( \left( \frac{\partial \tilde{\phi}}{\partial \mu_L} \mu_L + \frac{\partial \tilde{\phi}}{\partial \mu_H} \mu_H \right) / \frac{\partial \tilde{\phi}}{\partial \mu} = 0 \), or assuming that \( \frac{\partial \tilde{\phi}}{\partial \mu} |_{\phi = 0} \neq 0 \); \( \frac{\partial \tilde{\phi}}{\partial \mu_L} \mu_L + \frac{\partial \tilde{\phi}}{\partial \mu_H} \mu_H = 0 \). Since we are using the Normal approximation for the Poisson distribution,

\[
F_r(s_r(v)) = \frac{1}{2} \left[ 1 + erf \left( \frac{s_r(v) - \mu_r l}{\sqrt{2\mu_r l}} \right) \right] = \frac{1}{2} + \frac{1}{2\sqrt{\pi}} \int_{s_r(v) - \mu_r l}^{\infty} e^{-t^2} dt = 1 - \frac{c}{v}.
\]

We now define \( z_r = \frac{s_r(v) - \mu_r l}{\sqrt{2\mu_r l}} \) and note that \( z_H = z_L = z \), since \( F_H(s_H(v)) = F_L(s_L(v)) = 1 - \frac{c}{v} = \Phi(z) \), where \( \Phi(\cdot) \) is the cumulative distribution function for the Standard Normal distribution. Therefore: \( \frac{\partial F_r(s_r(v))}{\partial \mu_r} = e^{-\frac{(s_r(v) - \mu_r l)^2}{2\mu_r l}} \frac{\partial}{\partial \mu_r} \left( \frac{s_r(v) - \mu_r l}{\sqrt{2\mu_r l}} \right) = \frac{\partial}{\partial \mu_r} \left( 1 - \frac{c}{v} \right) = 0 \). This gives us the result that \( \frac{\partial z}{\partial \mu_r} = 0 \). Furthermore, it is a well-known result that for the Standard Normal distribution \( \int z \Phi(z) dz = z\Phi(z) + \phi(z) \), where \( \phi(\cdot) \) is the density function of the Standard Normal. After a little algebra, we obtain \( \int_{s_r(v)}^{\infty} (1 - F_r(x))dx = (\frac{c}{v} - \Phi(z)) \sqrt{\mu_r l} \). Now using the result that \( \frac{\partial \tilde{\phi}}{\partial v} = 0 \) and the fact that \( f_r(s_r(v)) = \phi(z)/\sqrt{\mu_r l} \), it is straightforward to verify that at \( \tilde{\phi} = 0 \) we have \( \frac{\partial \tilde{\phi}}{\partial \mu_L} \mu_L + \frac{\partial \tilde{\phi}}{\partial \mu_H} \mu_H = 0 \), thus implying that \( \frac{dv}{dQ} = 0 \). □

Proof of Proposition 7: Under PBC, the payoffs for the vendor (selling to \( k \) buyers) and an individual buyer become \( V_r^k = kw_r - vE_r(B|s_r(v_r,k)) - cs_r(v_r,k) - k\mu_r M \) and \( U_r = -w_r + \frac{w_r}{k} E_r(B|s_r(v_r,k)) - \mu_r r \), where \( s_r(v_r,k) \) is the inventory chosen by the vendor when the penalty rate is \( v \) and there are \( k \) buyers. The quantity \( E_r(B|s_r(v_r,k)) \) represents the total number of expected backorders caused by all \( k \) buyers. Due to symmetry, each buyer experiences a \( k^{th} \) fraction of these backorders. The characterization of the vendor’s outcome in the symmetric equilibrium with inventory pooling across \( k \) buyers proceeds in exactly the same way as in Proposition 5; the only difference is that the failure rate is \( k\mu_r \) instead of \( \mu_r \) for \( \tau \in \{L, H\} \). Importantly, the optimal penalty \( v_H^* \) is that given by \( (4) \). In order to establish that the \( H \)-type vendor’s payoff improves when she has the ability to pool inventory across buyers, we must show that \( \forall k \geq 1, V_H^{k*} > kV_H^* \); that is, the vendor achieves a higher payoff by pooling inventory across \( k \) buyers than in the absence of pooling. A necessary and sufficient condition for this to be true is \( V_H^*(N) > NV_H^*(1), \) where
$V_H^*(i)$ is the payoff of the $H$-type vendor when contracting with $i$ buyers, each purchasing a single engine. The latter condition is equivalent to treating $N$ as a continuous variable and verifying that \( \frac{d}{dN} \left( \frac{V_H^*(N)}{N} \right) > 0 \). We now introduce an additional notation $\mu_r(1)$ denotes the failure rate of one engine of type $\tau$, and therefore $\mu_r = N \mu_r(1)$. Using the result in Proposition 5 we know that

\[
V_H^*(N) = (v_H^* - \chi)p \left[ E_L(B|s_L(v_H^*,N)) - E_H(B|s_H(v_H^*,N)) \right] - \chi E_H(B|s_H(v_H^*,N)) - cs_H(v_H^*,N) - N(\mu_H(1)M + \mu_p(1)r + \theta).
\]

We define a function $g(N)$ such that $V_H^*(N) = g(N) - N(\mu_H(1)M + \mu_p(1)r + \theta)$. Then

\[
\frac{d}{dN} \left( \frac{V_H^*(N)}{N} \right) > 0 \iff \frac{dg(N)}{dN} > \frac{g(N)}{N}.
\]

Using the Normal approximation to the Poisson distribution, $g(N)$ can be expressed as:

\[
g(N) = L(z) \left( (v_H^* - \chi)p(\sqrt{\mu_l} - \sqrt{\mu_H}) + \chi \sqrt{\mu_H} \right) - cs_H(v_H^*,N),
\]

where $L(z) = -z(1 - \Phi(z)) + \phi(z)$; $\Phi$ and $\phi$ being the distribution and density functions respectively, for the Standard Normal distribution, $v_H^*$ is the solution to (4), $z = \frac{s_H(v_H^*,N) - \mu_H}{\sqrt{\mu_H}}$, and $s_H(v_H^*,N)$ is the solution of $F_H(s_H(v_H^*,N)) = \left( 1 - \frac{c_H}{v_H^*} \right)$. As shown in Lemma 2 (and in its proof),

\[
\frac{d}{dN} = 0, \quad \frac{d}{dN} = \frac{\mu_r}{N}, \quad \text{and} \quad \frac{d}{dN} = 0.
\]

Using these relationships, we can show that

\[
\frac{d}{dN} = \frac{g(N) - 2c_H l}{2N}.
\]

Then

\[
\frac{d}{dN} > \frac{g(N)}{N} \iff g(N) + 2c_H l < 0.
\]

This can be rewritten as $V_H^*(N) + N(\mu_H(1)M + \mu_p(1)r + \theta) + 2c_H l < 0$. Now using the expression for $V_H$ from Proposition 1, we recover the condition of Proposition 7. Finally, note that if pooling across $k$ buyers is possible, both types of vendor will always choose to pool inventory; not pooling is not credible (renegotiation-proof) as after the contract is signed, but before it is implemented, it is Pareto-improving to pool inventory. □

References


