Physics 510 – Fall 2014

Methods of Theoretical Physics

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Course Website: http://physics.usc.edu/~bars/510/

Homework: questions and answers

Lecture times: Mo. & Wed. 10:00 - 11:50 AM, Room: KAP 137.
Office hours: Mo. 1- 2 PM, or e-mail for appointment.

Course description: (for detailed outline of lectures see attachment)
Functions of complex variables; Gamma and Beta functions; Dirac delta function; Linear algebra, vector analysis; Hilbert space; Fourier and Laplace transforms; Infinite, asymptotic and Fourier series; Orthogonal polynomials; Bessel and Legendre functions; Spherical harmonics; Differential equations; Sturm-Liouville theory; Introduction to group theory.

Grading (See: The Trojan Integrity Guide and Guide for Avoiding Plagiarism)
Homework and class participation 30%; Midterm 30% (Wednesday, October 8, in class);
Final 40% (Monday, December 15, 8-10 AM, room: same as classroom)
Homework assigned on Monday or Wednesday is due the following week on Wednesday.

Lecture Notes: 1) C. Pope I, 2) C. Pope II, 3) G. Gibbons, 4) I. Bars

Textbooks:(for detailed description of these books see attachment)
M. Stone and P. Goldbart, "Math for Physics: A Guided Tour for Grad Students," Cambridge (2009).
M. Vaughn, "Introduction to Mathematical Physics," Wiley (2007).
J.J. Kelly, "Graduate Mathematical Physics," Wiley (2007).
G.B. Arfken and H.J. Weber, "Mathematical Methods for Physicists," Academic Press/Elsevier (2005)
M. Ablowitz and S. Fokas, "Complex Variables," 2nd edition, Cambridge University Press (2003).
P. Dennery and A. Krzywicki, "Mathematics for Physicists," Dover (1996).
F. Byron and R. Fuller, "Mathematics of Classical and Quantum Physics," Dover (1992).
P. Morse and H. Feshbach, "Methods of Theoretical Physics," McGraw-Hill (1953).
E. Whittaker and G. Watson, "The Course of Modern Analysis," Cambridge University Press (1927).
M. Abramovitz and I. Stegun (eds.), "Handbook of Mathematical Functions," NBS, AppMath Series 55.
I.S. Gradsteyn and I.M. Ryzhik, "Tables of Integrals, Series and Products," 7th edition,

Some useful links:
NIST Digital Library of Mathematical Functions
Resource for mathematics: Wolfram Math World
Useful links in Wikipedia article on Complex Analysis
Examples of solved problems in math and physics
Mathematica: a self-study course by J.J. Kelly; see also "Help" in Mathematica

LECTURES ON MATHEMATICAL METHODS OF PHYSICS
Itzhak Bars
Fall 2014

The following list of topics is a tentative plan. The actual topics covered in the lectures may be subject to change during the semester and not all topics will be covered in full detail. The material covered will be defined by lecture presentations, handouts and homework assignments.

Material that is similar to the lectures is covered in the book by Stone and Goldbart, as well as in the books by Vaughn or Dennery and Krzywicki (D&K) in the chapters or sections indicated. In preparing my lectures I will consult these as well as other references (see the lecture notes and books listed in the outline).

1 Complex Analytic Functions

(3-4 weeks, Chapter 4, from Vaughn; Chapter I from D&K)

1. Complex numbers 97
2. Analytic functions 98
3. The Riemann-Cauchy equations 100
4. Harmonic functions 101
5. Graphs of complex functions 104
6. Conformal mappings 104
7. Second look at elementary functions 106
8. Multivalued functions, Riemann surfaces and branch cuts 107
9. Complex integration 114
10. Cauchy and Cauchy-Goursat theorems, consequences 117
11. Taylor series expansion 123
12. Laurent expansion 125
13. Taxonomy of singularities 128
14. The residue theorem 129
15. Definite real integrals 131
16. Expansions of meromorphic functions 137

2 Generalized Functions

(1 week, or III-13 from D&K)

3 Vectors, Tensors, Matrices and Operators

(2 weeks, Sections 2.1-2.4 from Vaugn; Chap. II from D&K)

4 Hilbert Spaces

(2 weeks, Sections 7.1-7.5 from Vaughn, Chapter III from D&K)

5 Fourrier and Laplace Transforms

(1 week, Sections 6.3-6.4 from Vaughn, Chap. III-12 from D&K)

6. List of properties and examples of Fourier transforms 89
7. Apodization 92
8. Applications of Fourier transforms 93
9. Laplace transform 170
10. Inverse Laplace transform 172

6 Differential Equations

(2 weeks, Sections 5.1-5.4 from Vaughn, Chap. IV part I from D&K)

1. Singular points of second order eqs 154
2. The Frobenius-Fuchs theorem 157
3. Classification of basic equations: one singular point 160
4. Two singular points 161
5. Hypergeometric equation 164
6. Reductions of solutions to hypergeometric functions 167
7. Integral representation of hypergeometric functions 168
8. Confluent hypergeometric equation 169
9. Fuchsian equations 169
10. Generalized hypergeometric functions 169
11. Dealing with irregular singular points 170

6.1 Legendre & Bessel Functions, Orthog. Polynomials

(2 weeks, Section 6.5 from Vaughn, Chap III-10 from D&K)

1. Classical orthogonal polynomials
2. Rodriguez formula & classification
3. Legendre Eq.
4. Legendre Polyn
5. Associated Legendre
6. The Laplacian for Bessel Function 174
7. Bessel equation 175

8. Bessel functions of the integer order 178
9. Bessel functions and the Kepler problem 180
10. Asymptotics of Bessel functions 181
11. Other Bessel functions 182

6.2 Partial Differential Equations

(2 weeks, Sections 8.1-8.4 from Vaughn, Chap.IV part II from D&K)
Brief outline; insufficient time for 1-semester course

7 Introduction to Group Theory & Lie Groups

(3 weeks, Group Theory notes by I. Bars)

1. SU(2) Lie Algebra, Angular momentum & Spin
2. Rotation operators, $\exp(\textit{Lie algebra})$
3. Representation bases $|jm\rangle$, matrix representations of the Lie algebra
4. Spherical harmonics $Y_{lm}(\Omega)$ and associated Legendre polynomials
5. Rotation matrices $\mathrm{D}^{j}_{mn}(\Omega)$, representations of group elements

BOOKS FOR MATHEMATEMATICAL METHODS OF PHYSICS
Phys. 510

The books are listed in order of publication date (most recent first)

- M. Stone and P. Goldbart, "Mathematics for Physics: A Guided Tour for Graduate Students," Cambridge University Press (2009).
  An excellent textbook based on a two semester math methods course at the University of Illinois in Urbana-Champaign. It covers many topics in detail and includes additional material not covered in the one-semester class Phys.510. It has a lot of interesting examples and is very well written. A pre-publication copy can be downloaded from
  http://www.physics.gatech.edu/~pgoldbart6/PostScript/MS_PG_book/bookmaster.pdf

- M. Vaughn, "Introduction to Mathematical Physics," Wiley (2007).
  This textbook has been written for a one-semester course in mathematical methods. In addition it contains a good exposition of some topics that are usually not covered in traditional texts, such as introduction to differential geometry, advanced theory of Hilbert spaces and some introductory group theory, that you may find useful in the future.

- J.J. Kelly, "Graduate Mathematical Physics," Wiley (2007).
  A fairly concise new textbook, which makes an extensive use of Mathematica.

- G.B. Arfken and H.J. Weber, "Mathematical Methods for Physicists," 6th edition, Academic Press/Elsevier (2005).
  This has been a traditional textbook for graduate courses in mathematical methods, but it is quite diffcult to use for a one semester course covering only a selection of topics that are discussed in this book. Indeed, given its size (1200 pages), the style and the amount of information, it is really more an encyclopedia than a typical textbook. You should become familiar with it and learn how to use it as a reference! Try to get hold of any edition (4th or 5th or 6th).

- M. Ablowitz and S. Fokas, "Complex Variables," 2nd edition, Cambridge University Press (2003).
  This is as pedagogical as the classic older textbook on complex variables by Churchill and Brown, but it covers more material, both theory and applications, and therefore seems to be a better investment.

- P. Dennery and A. Krzywicki, "Mathematics for Physicists," Dover (1996).
  Another good old textbook originally published in 1967.

- F. Byron and R. Fuller, "Mathematics of Classical and Quantum Physics," Dover (1992).
  A textbooks originally published in 1969/70. It has a good, concise exposition of the theory of Hilbert spaces, analytic functions and integral equations.

- P. Morse and H. Feshbach, "Methods of Theoretical Physics," McGraw-Hill (1953).
  An old classic, which is a gold mine of information about special functions and differential equations in physics.

- E. Whittaker and G. Watson, "The Course of Modern Analysis," Cambridge University Press (1927).
  An excellent textbook on special functions, first published in 1902.

- M. Abramovitz and I. Stegun (eds.), "Handbook of Mathematical Functions," National Bureau of Standards Applied Mathematics Series 55.
  An extremely useful collection of de ntions and properties of special functions presented in a user friendly format.
  Available on-line at http://people.math.sfu.ca/~cbm/aands/ .

- I.S. Gradsteyn and I.M. Ryzhik, "Tables of Integrals, Series and Products," 7th edition, A. Jeffrey and D. Zwillinger (eds.), Academic Press/Elsevier (2007).
  Classic tables that used to be indispensable for any serious physicist. These days much of the information contained both in this book (and in Abramowits and Stegun) has been implemented into routines of algebraic manipulation programs such as Maple, Matlab or Mathematica.

- E. Weisstein and others, "Wolfram MathWorld," at http://mathworld.wolfram.com.
  An extremely useful on-line mathematics resource. If you are looking for a precise definition or properties of some special functions, this may be the place to start.

Methods of Theoretical Physics, Fall 2014
Homework and Exams, Questions and Answers

Homework assigned on Monday or Wednesday is due the following week on Wednesday
The presentation must be well organized, include comments and be clearly legible.

Students are encouraged to consult any resources and to discuss among themselves while preparing the homework. However, each student must write his/her own version of the homework. It should not be copied directly from some external resource or from some other student. Identical presentations by different students will be graded 0 points.

Students are encouraged to use Mathematica to both learn how to use it and to check their answers. However, the student must explain how the answer to a problem is obtained and provide the steps – reporting only an answer given by Mathematica or another resource is not acceptable as the solution to homework assignments.

Exam problems will be based on material presented in lectures, handouts, homework assignments. Some exam problems may be variations of homework problems.

Homework 1: Questions at this link. Answers at this link.
Homework 2: Questions at this link. Answers at this link.
Homework 3: Questions at this link. Answers at this link.
Homework 4: Questions at this link. Answers at this link.
Homework 5: Questions at this link. Answers at this link.
Homework 6: Questions at this link. Answers at this link.
Homework 7: Questions at this link. Answers at this link.

Midterm : Wednesday, October 8. (30% of grade)
Time: 10:00-11.50. Room: same as classroom.
This is a closed book and closed notes exam. You are allowed to bring 1 sheet of notes.
Questions and Answers

Homework 8: Questions at this link. Answers at this link.
Homework 9: Questions at this link. Answers at this link.
Homework 10: Questions at this link. Answers at this link.
Homework 11: Questions at this link. Answers at this link.
Homework 12: Questions at this link. Answers at this link.
Homework 13: Questions at this link. Answers at this link.

Final: Monday, December 15 - (40% of grade)
Time: 8:00 AM -10:00 A.M,. room: same as regular classroom
This is a closed book and closed notes exam. You are allowed to bring 2 sheets of notes.
Questions and Answers