MATH 410 FALL 2014

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Office Hours: 2 – 3:15 MW, Tu 1:15 – 3:30, and by appointment.

Course hours: 1:00 – 1:50 MWF in KAP 134 *and* Th 2:00 – 3:50 in KAP 245 (enter via Math Center)

Text: *Concepts in Abstract Algebra* by C. Lanski ($89 new from: www.ams.org).

Grading: Homework, two midterms (on Thursdays 10/2 and 11/6), and the final exam (Wednesday, 12/17, from 11AM – 1PM) each counts for 25% of the course grade. Half of the letter grade on the final exam may replace half of the letter grade on one of the other three graded parts of the course grade.

Prerequisite: Math 225. Matrix calculation, mostly for 2×2 or 3×3 matrices, will be used for examples. Also, the fact that Math 225 deals with some abstraction is important as an introduction to abstraction here.

Material of the Course and Other Goals

We start with a brief review of some fundamental notions (a bit of Ch. 0, most of Ch. 1), then about half the course concerns group theory (parts of Ch 2 – Ch 8), and the remainder of the course is an introduction to rings and fields (from Ch. 10, 11, and maybe 12). The goals of the course are to learn the material presented, to learn to deal with mathematical notation and abstraction, to learn to think precisely and logically, and to learn how to make mathematically valid arguments (i.e. construct proofs!).

General Comments

You are responsible for the material presented or assigned in class, including homework. It is important to study the text to get a better understanding of the material than is possible from the lectures alone. The text provides a fuller, richer account of the material than can be presented in class, and also provides more examples than can be done in class. It is also important to work on problems. The problems are based on the text material and their purpose is to help you to learn that material, so *study the text* to see what material is relevant for the problem at hand; the more problems you work on, the better you will learn the material. A main difference here from other math courses is the focus on definitions, theorems, their relations, and their uses, rather than on computation and procedures for solving problems. Therefore, it is likely that you need to study differently from what you may be used to in prior math courses.

How to Study the Material

Math 410 is difficult for most students for three related reason: many new terms and ideas presented, constructing proofs is a requirement, and one must develop habits of precision to deal with the first two problems—both to learn exactly what the new material is and to use this material effectively. To assure precision in understanding and using the results and concepts, one must *memorize* the definitions and theorems, not always essential in previous courses. Statements of definitions and theorems are typically asked for on exams. Proofs of theorems may also be asked for if the arguments are short and straightforward. Knowing definitions is vital: One cannot understand the meaning of the theorems or see how to solve most problems without knowing precisely what the words in these statements mean. Knowing the theorems is essential: Constructing mathematically acceptable arguments is an art unfamiliar to most students and is usually *impossible* without knowing the theorems and definitions.

The best way to memorize and to understand the definitions and theorems is to look at them directly, to work on problems, and to study proofs in the text. The problems give practice with the concepts and techniques, and illustrate the use of the theorems, so help you to remember the results proved in the course. Problems also provide experience in constructing proofs. Thus, the purpose of the problems is to help you to learn the material and associated techniques; they are not to be done using basic notions unrelated to the course material. Attack these problems by studying the material in that section and in earlier ones, as needed. Finally, studying proofs helps to see how definitions and other results are used, so helps to remember and understand them; it is easier to understand material in context than by itself. Also studying proofs helps to see what constitutes a proof.

Constructing Proofs

Constructing a proof typically requires a *complete command* of the definitions and theorems. Unless you understand exactly what is to be proved, it is difficult to get started, and knowing the definitions precisely may itself lead to a correct solution of the problem. A proof is a sequence of statements, each true for some identifiable reason; a proof should connect in steps what you are given with what you want to conclude. *If you cannot identify a specific definition or theorem to justify a statement you make in a proof then there is a very good chance that your argument is not correct.*

To start working on a proof it is vital to have a clear understanding of what must be proved and then to ask yourself how that could be shown. It may be that it is only necessary to verify a definition. For many problems it may be difficult to see how to obtain the conclusion from the assumptions (hypothesis) directly or easily. In this case it may be that what you need to prove is the conclusion of a known theorem—so you need to know previous results. To apply that theorem you need to show its hypothesis holds. That hypothesis may be what you are given in the problem at hand, may follow from a definition, or may hold by applying another theorem whose hypothesis is what is given in the problem. Continuing in this way, one works *backwards* until it is clear how the hypothesis of the given problem can be used to connect the parts of this chain of reasoning. The formal written version of the argument is given by proceeding forward from the hypothesis.

When you write out solutions to the problems, you must give reasons for the statements made, with clear reference to results proved or to definitions, where appropriate. Use the approach of the text as an example of how to do this. Providing reasons for statements is good practice for learning to think carefully and precisely, is more likely to lead to a correct solution, and reinforces the learning of theorems and definitions.

Advice/Help

Learning to deal with abstraction and learning to write proofs are initially difficult tasks for most students, but will become much easier *if* you spend the time necessary and *if* you memorize the definitions and theorems. Since this is a substantial upper division course, you may need to spend considerable time to master the material. Also, lectures will make more sense if you keep up with the material. Much of what we will do is cumulative and each lecture assumes that material presented previously is (at least a bit) familiar to you.

You likely will find some of the homework problems to be difficult and you may not see what approach to use. Please feel free to come to see me, *frequently* if necessary. I will gladly give you help and hints. I typically try to help you to see how to think about the problems rather than just telling you what to do (Socratric method!), with the hope that you will eventually learn how to attack problems yourself. Of course, you will not be able to ask me questions about the homework if you look at the problems for the first time the night before they are due! It is very difficult and inefficient to discuss mathematics via e-mail: face-to-face meetings are best for such discussions. Also note that I do not usually check and respond to e-mail other when I am in my office. If you have any problems with the material itself or with the homework, or if you want to discuss *anything* else or just chat, come in to see me. I will make every effort to make myself available at your convenience. I am here to help you however I can. Finally, in my (long) experience with this interesting and challenging course, everyone who takes the course seriously gets through it satisfactorily, and some students do extremely well in it. It may take some effort, and weeks, to feel more comfortable dealing with the formal and abstract approach to the material—one of the goals of the course, but try to be patient, keep working on it, and come to see me about it—you will succeed!

Academic Integrity

The exams are all "closed book" and when taking them you may not use notes, consult with others, or copy from others. Since the collected homework problems count toward the course grade, the work you turn in as homework is to be *yours alone*. You may ask me or the TA for hints, and you may discuss general approaches with others in the course, but otherwise *for those homework problems to be handed in for grading, you may not copy solutions from anyone else or any other source (e.g. the internet)*: *this includes the faculty and TAs at the Math Center*. Note that "copy" includes changing notation or the order of steps. Violation of these policies is a serious matter.